Mañé’s Theorem for entire functions and its applications

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$f$ is called semi-hyperbolic at a point $x$ in the Julia set $J_f$ of $f$ if there exists a
neighborhood $U$ of $x$ and an $N \in \mathbb{N}$ such that for any connected component $V$ of
$f^{-n}(U)$ ($\forall n$), $f^n|_V : V \to U$ satisfies

$$\text{deg}(f^n|_V : V \to U) \leq N.$$ 

In the case that $f$ is transcendental, we add the following property:

$$f^n|_V : V \to U \text{ is proper for every } V.$$ 

Mañé’s Theorem for rational functions ([2]) asserts that if $f$ is a rational function
and $x \in J_f$ satisfies

$$x \notin \bigcup_{c \in \text{Rec} \cap J_f} \omega(c) \cup \{\text{parabolic periodic points}\},$$

where $\text{Rec} = \{\text{recurrent critical points}\}$, then $f$ is semi-hyperbolic at $x \in J_f$.
Actually the converse of this theorem is also true. On the other hand, in the case
that $f$ is transcendental, a new obstruction for semi-hyperbolicity is known ([1]).
In this talk, we show a necessary and sufficient condition for semi-hyperbolicity for
an entire function $f$. Also we show some results on measure theoretical properties
of the dynamics of (transcendental) entire functions as an application.