Applications of the filtered Poincaré lemma in higher level

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This work [5] is part of a program to use the partial divided powers of Berthelot [1] to get new insight into the $p$-adic cohomological theories for algebraic varieties of positive characteristic. Unlike other works ([3], [2]), we do not use crystalline cohomology of higher level as a tool to obtain results in rigid cohomology and, in particular, we do not ignore torsion, which is in fact very rich in this theory.

We start by reinterpreting the de Rham complexes introduced in [4] as particular cases of what we call a Berthelot-Lieberman complex, and use this to prove the filtered Poincaré lemma for transversal crystals.

Independently, we apply the same general construction to define the conormal complex of higher level of a group scheme and to study its relation to invariant differential forms of higher level. Unlike the classical case (level 0), not all invariant forms are closed. Actually, the module of closed invariant forms is isomorphic to the first cohomology group of the conormal complex. We present concrete examples and give the relation with de Rham cohomology of higher level in the case of abelian schemes.

Putting together these two instances of the Berthelot-Lieberman construction, we show that crystalline extensions of transversal crystals by algebraic groups can be computed by reduction to filtered de Rham complexes. As an application, we show that the extension group of the partial divided power ideal by a smooth group is nothing else but a lifting of the module of closed invariant differentials of higher level.