An invariant for Stallings manifolds from a TQFT

P. Semião

Departamento de Matemática, Universidade do Algarve, Faculdade de Ciências e Tecnologia 8000-062 Faro, Portugal [psemiao@ualg.pt]

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We obtain an invariant of Stallings manifolds using a Topological Quantum Field Theory (TQFT) approach. The TQFT’s are specified by matrix equations, which are solved using unitary matrices. In our approach a TQFT is a monoidal functor from a “topological” monoidal category to an “algebraic” monoidal category. The objects of the topological category are triples \((X, A, m)\), where \(X\) is an orientable connected 3-dimensional manifold with boundary, \(A\) is an orientable connected 2-dimensional manifold without boundary, and \(m : A \to X\) is an “inclusion” morphism. The morphisms of this category are isomorphisms and gluing morphisms. On the algebraic side, the objects of the category are pairs \((V, x)\), where \(V\) is a vector space over a field with an involution, and \(x\) is an element of \(V\). The morphisms are linear maps which preserve the elements of the respective spaces. The TQFT functor preserves an additional structure given by two monoidal endofunctors, corresponding to change of orientation on the topological side and changing the scalar multiplication using the involution on the algebraic side.

A fundamental feature of TQFT is the gluing of two spaces, and the present approach describes this operation in terms of morphisms in the topological category, but describes equally well the self-gluing of a single space. E.g. if \(S\) is a 2-dimensional manifold without boundary endowed with a fixed orientation and \(\varphi : S \to S\) is an orientation-preserving automorphism, then the self-gluing of the cylinder \(S \times I\), where \(I\) is the standard closed unit interval, is a 3-dimensional manifold \(S_\varphi := \frac{S \times I}{\sim_\varphi}\) known as a Stallings manifold, where \(\sim_\varphi\) is the relation generated by the relation \((x, 0) \sim (\varphi(x), 1)\).