Partial differential equations related to generating functions

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The Faber polynomials $(F_k)_{k\geq 1}$ are given by the identity

$$1+b_1w+b_2w^2+\cdots+b_kw^k+\cdots=\exp\big(-\sum_{k=1}^{+\infty}\frac{F_k(b_1,b_2,\ldots,b_k)}{k}\,w^k\big)$$

The polynomials $(K_n^p)_{n\geq 1}$, $p\in Z$ are given by

$$(1+b_1w+b_2w^2+\cdots+b_kw^k+\cdots)^p=1+\sum_{n\geq 1}K_n^p(b_1,b_2,\ldots,b_n)w^n$$

They are homogeneous polynomials of degree n in the variables $(b_1,b_2,\ldots)$ where b_k has weight k. Let $X_0=-\sum_{j\geq 1}b_j\frac{\partial}{\partial b_j}=-b_1\frac{\partial}{\partial b_1}-b_2\frac{\partial}{\partial b_2}-\cdots-b_k\frac{\partial}{\partial b_k}-\cdots$. Then the identity

$$K_n^{-1}=\sum_{0\leq j\leq n}K_j^{-2}K_{n-j}^1$$

which correponds to the multiplication $h(z)^n=h(z)^{n-1}h(z)$ is the same as the partial differential equation on the manifold of coefficients

$$\frac{\partial^2F_n}{\partial b_r\partial b_s}=\frac{\partial}{\partial b_{r+s}}(X_0F_n)\qquad\forall r,\ s\geq 1,\quad n\geq 1$$

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