Real interpolation of Banach algebras

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2000 Mathematics Subject Classification. 46B70, 46M35, 46H05

A classical result of A.P. Calderón [2] shows that the Banach-algebra structure is stable by complex interpolation. For the case of the real method $(\cdot,\cdot)_{\theta,q}$, it is known that $(\cdot,\cdot)_{\theta,1}$ does it. Even the more general methods $(\cdot,\cdot)_{\rho,1}$ have this property, as was proved by Zafran [5] and Kaijser [4]. Recently, Blanco, Kaijser and Ransford [1] have shown another class of real interpolation methods which preserves also the Banach-algebra structure.

We shall work with the general real method $(\cdot,\cdot)_\Gamma$ which is defined by replacing the usual weighted $L_q$ norm of $(\cdot,\cdot)_{\theta,q}$ by a more general lattice norm $\Gamma$, and we shall show that a necessary and sufficient condition for the method $(\cdot,\cdot)_\Gamma$ to respect the Banach-algebra structure is that $\Gamma$ be a Banach algebra with multiplication defined as convolution. Combining our result with interpolation properties of weakly compact operators one can derive the result of Blanco, Kaijser and Ransford [1] on factorization of weakly compact homomorphisms between Banach algebras.

These results are part of the joint paper [3] with F. Cobos and A. Martínez.