Applications of the Loewner equation to crack propagation in brittle solids

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2000 Mathematics Subject Classification. 74R10, 74B05, 74G70, 30C55, 30C70

A. A. Griffith in [1] laid the foundations of the modern approach to brittle fracture. He considered the rupture problem as a competition between two different energies involved during crack growth: The stored elastic energy released (reversible) and the crack surface energy (dissipative). He was able to write one scalar equation showing the condition of critical equilibrium for a crack configuration. In a two dimensional setting at least two scalar relationships are needed to determine the subsequent crack path.

In this work we study the propagation of a crack in critical equilibrium for a brittle material in a Mode III field. The energy variations for small virtual extensions of the crack are handled in a novel way: the amount of energy released is written as a functional over a compact family of univalent functions on the upper half plane. Classical techniques developed in connection to the Bieberbach Conjecture are used to quantify the energy-shape relationship. We apply Schiffer’s boundary variation technique (see [2]) to find optimal paths in the sense of maximum elastic energy released. By means of a suitable parameterization generated by the Löwner equation we impose a stability condition on the field which derives in a local crack propagation criterion. We called this the anti-symmetry principle, being closely related to the well known symmetry principle for the in-plane fields.
