An embedded minimal surface with infinite total curvature: a Jenkins–Serrin problem on an unbounded domain

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The topic of minimal surfaces in flat 3-manifolds with finite genus but infinite total curvature has recently attracted some attention [1, 2]. In the complete flat 3-manifold $\mathbb{R}^2 \times S^1$, the only known examples of properly embedded minimal surfaces with infinite total curvature but finite genus come from doubly periodic minimal surfaces in $\mathbb{R}^3$. In particular, they are all periodic examples in $\mathbb{R}^2 \times S^1$.

Karcher [4] constructed, as an application of a theorem by Jenkins and Serrin [3], properly embedded minimal surfaces in $\mathbb{R}^2 \times S^1$ with genus zero and finitely many Scherk-type ends, which he named Saddle Towers. We follow a similar idea to obtain non periodic properly embedded minimal surfaces in $\mathbb{R}^2 \times S^1$ with genus zero, infinitely many ends and exactly one limit end (in particular, they have infinite total curvature). Moreover, each one of such surfaces is asymptotic to either two doubly periodic Scherk minimal surfaces or a Toroidal Halfplane Layer [4, 5, 6], which have recently been classified in [6] as the only properly embedded doubly periodic minimal surfaces in $\mathbb{R}^3$ with genus one and finitely many parallel Scherk-type ends in the quotient.