Using envelopes to construct reflecting and refracting profiles

N. Alamo, C. Criado*

Departamento de Algebra, Geometría y Topología, Universidad de Malaga, 29071 Malaga, Spain,
Departamento de Física Aplicada I, Universidad de Malaga, 29071 Malaga, Spain
[c.criado@uma.es]

2000 Mathematics Subject Classification. 78A05

For any wave front $W$ and any source point $F$ not in $W$, it is possible to construct a family of reflecting profiles $\{M_a\}$ parametrized by a non-negative real number $a$, with the property of producing the given wave front $W$, after reflecting in each profile $M_a$ a spherical wave front produced in the point source $F$. If light propagation direction is reversed, then the same family of mirrors has the property of focusing the wave front $W$ in the point $F$. The construction of the profiles $M_a$ for the wave front $W$ and the point $F$ proceeds by taking the envelope of a family of quadrics $\{h_a(x)\}_{x \in W}$ (hyperboloids and ellipsoids of revolution). Basically, the quadrics are those with foci $F$ and a point $x$ varying in the wave front $W$, and eccentricity $e = \frac{|x-F|}{2a}$; $2a$ is the distance between the vertices of $h_a(x)$. The profiles $M_a$ are smooth surfaces except for the points which are centers of curvature of the wave front $W$ [1]. To construct refracting profiles we use a similar method but using Descartes ovals of revolution instead of quadrics [2]. The proposed methods can be useful in the construction of optical, radar, or acoustical devices, and have other interesting applications [3] [4].