On the transversal curves in the Dulac-Cherkas function construction for the number of limit cycles estimation

Aliaksandr Hryn

Department of mathematics and informatics, Yanka Kupala State University of Grodno, Ozheshko str. 22-228, Grodno, 230023, Belarus [grin@grsu.by]

2000 Mathematics Subject Classification. 34C07, 34C05, 37G15

In the focus of the report lies the elaboration of algorithms for the construction of Dulac-Cherkas functions for the planar dynamical systems

$$\frac{dx}{dt} = P(x, y), \quad \frac{dy}{dt} = Q(x, y), \tag{1}$$

with polynomial P and Q (the second part of the 16-th Hilbert's problem) as well as with non-polynomial right part in some region $\Omega \subseteq R^2$ when $P, Q \in C^2(\Omega)$ in order to derive upper bounds for the number of limit cycles of (1) and to localize their position.

For this purpose the new approach of L. Cherkas [1] on usage of discontinuous Dulac function in the form of $B = |\Psi(x, y)|^{\frac{1}{k}}$, $k \neq 0$, $\Psi(x, y) \in C^1(\Omega)$, is developed, for which the inequality

$$\Phi \equiv D(\Psi) \equiv k\Psi divf + \frac{\partial \Psi}{\partial x} P + \frac{\partial \Psi}{\partial y} Q > 0 (< 0),\ \forall (x, y) \in \Omega,\ f = (P, Q) \tag{2}$$

holds. From the condition (2) follows the transversality of the curve $\Psi(x, y) = 0$, which decomposes the region Ω into subregions, in each of them the function B is the classical Dulac function, and the function Ψ would be called the Cherkas function. Then the number of limit cycles of the system (1) in the region Ω doesn't exceed the maximum number of non-homotopic to each other doubly-connected subregions, made by curves $\Psi = 0$ and $\partial\Omega$.

Such approach allows to work out a number of the effective methods of the localization and estimationof the number of limit cycles of the system (1) in bounded as well as in non-bounded regions Ω [2] [3].

The main attention in the report will be paid to the case, when the condition (2) isn't fulfilled, but the curve $\Phi = 0$ presents the transversal for the vector field of the system (1).

[1] Cherkas, L. A., Dulac function for polynomial autonomous systems on a plane, *Differential Equations* **5** (1997), 692–701.

[2] Grin, A. A., Cherkas, L. A., Extrema of the Andronov-Hopf function of a polynomial Lienard system, *Differential Equations* **41** (2005), 50–60.

[3] Cherkas, L. A., Grin, A. A., Spline approximation in the problem of estimating the number of limit cycles of autonomous systems on the plane , *Differential Equations* **42** (2006), 1–8.