Asymptotic behavior of smooth solutions for partially dissipative hyperbolic systems and relaxation approximation

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We study two problems related to hyperbolic systems with a dissipative source.

In the first part, we consider the asymptotic time behavior of global smooth solutions to general entropy dissipative hyperbolic systems of balance law in $m$ space dimensions, under a coupling condition among hyperbolic and dissipative part known as the Shizuta–Kawashima condition. Under the assumption of small initial data, these solutions approach constant equilibrium state in the $L^p$-norm at a rate $O(t^{-\frac m2(1-\frac 1p)})$, as $t \to \infty$, for $p \in [\min \{m, 2\}, \infty]$. The main tool is given by a detailed analysis of the Green function for the linearized problem. If the space dimension $m = 1$ or the system is rotational invariant, it is possible to give an explicit form to the main terms in the Green kernel.

In the second part, we consider the hyperbolic limit of special systems of balance laws: this means to study the limit of the solution to a system of balance laws under the rescaling $(t, x) \mapsto (t/\varepsilon, x/\varepsilon)$, as $\varepsilon \to 0$. For some special dissipative systems in one space dimension, it is possible to prove the existence of the limit and to identify it as a solution to a system of conservation laws.