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Short Communications Abstracts

Section 18

Applications of Mathematics in the Sciences

Dynamics of R&D investment strategies in Cournot competitions

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We present new and simple deterministic and stochastic dynamics on the production costs of Cournot competitions, determined by R&D investment strategies with and without uncertainty. At each period of time, the Cournot competition with R&D investment programs consists of two subgames. In the first subgame, both firms have initial production costs and choose R&D investment strategies, either with or without complete information, to obtain new production costs. The second subgame is a Cournot competition with parameters determined by the R&D investment program. We prove that the game presents either one, two or three Nash investment equilibria in the parameter regions studied. The Nash investment equilibria vary continuously with the initial production costs and with the differentiation of the goods. The deterministic dynamics, period after period, on the production costs of the duopoly competition appear from the firms deciding to play the Nash investment equilibria in the Cournot competition with R&D investment programs. Curiously, we prove that there is a piecewise smooth curve of stable equilibria which is robust under small parameter perturbations. We analyse the loss in the profits of one firm, if this firm decides not to invest in R&D. The stochastic dynamics on the production costs of the firms in a duopoly competition appear if we consider incomplete information in the R&D investment programs. We observe that the uncertainty deviates the mean of the stochastic trajectories from the deterministic trajectories of the production costs.

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To numerical modeling of the nonlinear reaction diffusion systems

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The properties of weak solutions of initial value problem for nonlinear reaction diffusion system which described by the degenerate type quasilinear parabolic equation and its systems at the presence of convective transfer, the volumetric absorption or source are studied. It supposed that the velocity of convective transfer is time-dependent. The method of investigation of new properties of solutions of initial value problem for reaction diffusion systems based on the nonlinear splitting is offered [1-3]. The conditions of existence of different type solutions and its estimates,asymptotes are established. The influence of the action of convective transfer and absorption (source) simultaneously to the velocity of the distribution of waves and fronts is analyzed. It is shown, that the action of the convective transfer and absorption (source) may be reduced to new effects such as an arising of "a wall" for a front, localization of the bounded and unbounded weak solutions. The estimates, asymptotes of the solutions and fronts of different type weak solutions depending on the value of parameters are obtained. It is proved a convergence of the self similar solution to the solution of Cauchy problem in special norm. Investigation of qualitative properties of solution of initial value problem, estimates of solution allowed to carry out the numerical experiments and visualization of studying reaction-diffusion processes. It is shown that offered methods successfully may be used for problem reaction diffusion with different kind of nonlinearity.

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Study of two different Riemann solvers for the lid-driven cavity flow

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An algorithm to solve the incompressible Navier-Stokes on a Cartesian Grid is presented. This algorithm is based on the Artificial Compressibility Method [1] and uses the cell-centered finite volume approach. A Godunov-type high order upwind scheme is applied to compute the flux at the cell interfaceses, involving polynomial reconstruction and the solution of a Riemann problem. The HLL Riemann Solver [2] and Roe's Riemann Solver [3] are implemented as part of the Godunov-type upwind scheme and results comparing their performance are presented.

To compute the optimal value for the coefficient of artificial compressibility in the case of the lid-driven cavity flow a grid refinement study is made for both Riemann Solvers.

The authors have compared by the rates of convergence and computational cost using HLL and Roe's Riemann Solver over a range of Reynolds numbers. The vortex location produced with each mboxRiemann Solver have been compared with published results [4].

This work is part of a project to use Artificial Compressibility Method with a Cartesian Cut Cell Method [5] and a free surface approach to simulate ship generated waves.

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Wavelet and block singular value image denoising

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This is a joint work with Akira Morimoto, Yuichi Shimano and Rémi Vaillancourt.

Denoising image data has been an active area of research with several different approaches being proposed using techniques such as wavelet thresholding, bilateral filtering and non-linear filtering based on singular value decomposition (SVD) [2]. A spline block singular value denoising method has been proposed in [3], as an improved version of the block singular value denoising found in [1].

A new hybrid method consisting of a discrete wavelet transform and a spline block SVD denoising procedure is proposed and used to remove Gaussian noise from images. Noise filtering is performed in the singular value and singular vector domains as follows: apply a two-dimensional discrete wavelet transform to a given image to get one approximation and three details; then apply a spline block singular value denoising with a spline weighting function to the approximation and apply a spline block singular value denoising with straight lines to the three details; finely, apply the two-dimensional inverse discrete wavelet transform to the processed approximation and the processed details to get the denoised image. A priori knowledge of the noise variance is not required because an estimate of the singular value noise variance is performed during the first phase of the procedure. Filtering is based on eliminating changes in singular values and singular vectors caused by additive Gaussian white noise or other types of noise. Processing the image in smaller blocks makes the SVD procedure computationally feasible.

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Coupled wave propagation in a rotating infinite random conducting magneto-thermo-viscoelastic medium

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This paper aims at discussing the problem of wave propagation in a rotating, randomly inhomogeneous, infinite interacting electrically conducting magneto-thermo-viscoelastic medium ([1]), smooth perturbation technique has been employed. The field equations, modified due to rotation, are put in the form $LV = f$, where L is a random linear operator, V the field vector and f is the non-random source term. Assuming

$$L = L_0 + \epsilon L_1 + \epsilon^2 L_2,$$

it can be shown that the mean field quantity $\langle V \rangle$, a seven-vector, representing the displacement, magnetic perturbation and thermal fields, satisfies an integro-differential equation involving the associated Green's tensor which has been computed. The electric conductivity is random but the thermo-mechanical coupling parameter is weakly random, that is, it is a random function proportional to ϵ, where ϵ measures the scale of random fluctuation of inhomogeneities from the non-random state with zero mean value. The thermal as also magnetic effects are discernible to ϵ^2 –order terms only. The relation connecting displacement amplitudes is presented. All cross-correlation functions between thermal and magnetic parameters disappear in the dispersion equation which has been deduced. Next, in order to study a particular case, both theoretically and numerically, all the correlation functions except the thermal coupling auto-correlation function are assumed to be zero. The terms representing thermal effects agree with those obtained by [2]. Next, the uncoupled dispersion equation involving longitudinal and transverse type waves affected by magnetic field alone is studied. The effect of Ω has been studied in detail. $k_{c,s}$ now depend upon Ω. The effect of the various perturbation terms appearing in the dispersion equation has been examined for high and low frequency fields. If λ, μ represent Lame's elastic moduli, all the results are valid for wave propagation in a magneto-thermoelastic medium when $\Omega \to 0$. Some numerical studies are being incorporated.

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Homogenization method in modelling of a heating of the faggoting metal in the transparent media

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Modern steelmaking involves a cycle of a hot working (annealing) of production in the protective atmosphere. Rolled bars come into annealing furnace in a packet condition, so the diameter D of the pocket is equal to 100 (and more) diameters d of single bars and a length of a bar is equal to 1000d and more. Besides, bars have periodic disposition in the faggot. These circumstances allow us to model a faggot hating as a unit one, based on the averaging theory. We shall neglect the convection and the thermal conductivity of the gas between bars. Then in any section of the faggot, which is orthogonal to its axis, boundary value problem of the heat exchange in a hard phase, looks as

$$c(u)\frac{\partial u}{\partial t} - \nabla_X(\lambda(u)\nabla_X u) = 0, \quad X \in \omega; \tag{1}$$

$$\left(\lambda(u)\frac{\partial u}{\partial \nu} + g(u)\right)|_P = \sum_i \int\limits_{\partial\omega_i'} g(u(Q_i))K(P,Q_i)dS_{Q_i}, \;\; P \in \partial\omega. \tag{2}$$

Here u is a temperature, $c(u)$ is heat capacity of a metal, $\lambda(u)$ - it's thermal conductivity; $X = (x_1, x_2)$ - vector of "slow" variables, $g(u) = \sigma u^4$ (σ is Stephen-Bolzman constant); ν is an exterior normal vector. An integrating in (2) is produced on those parts of boundaries $\partial\omega_i$ (adjoins to given section ω) which are visible from the point P. Function $K(P,Q_i)$ is a substantial part of elementary angle coefficients of radiation (see [1]), integrated along the length of the bar.

We shall find a solution of (1)-(2) as an asymptotic series $u \sim v_0(t,\xi) + \varepsilon v_1(t,X,\xi) + \ldots$, when $v_q(t,X,\xi)$ - periodic functions, depending from variables ξ_j. Here $\xi_j = \dfrac{x_j}{\varepsilon}$, when $\xi = (\xi_1, \xi_2)$ is a vector of "fast" variables, $\varepsilon = \dfrac{d}{D}$. So, basing on [2], we get, with the accuracy $O(\varepsilon^2)$, the equation describing a hate-transfer process within all the faggot:

$$c(v_0)\frac{\partial v_0}{\partial t} = \frac{\varepsilon}{2}\sum_{i,j=1}^{2} \alpha_{ij}\frac{\partial}{\partial x_i}\left(g_v'(v_0)\frac{\partial}{\partial x_i}\right),$$

where α_{ij} are not dependent on coefficients of a thermal conductivity $\lambda(v_0)$.

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Mathematical modeling and pseudospectral numerical simulation of genetic diseases in the human heart

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In recent years, the study of the electrical activity of the heart has evolved from a medical discipline to an interdisciplinary field that has caught the interest of mathematicians, physicists and engineers [1]. The work of such scientists has lead to a better understanding and characterization of cardiac arrhythmias, and our present knowledge about cardiac electrophysiology would not be the same without their contributions.

We present here a mathematical model created using only the minimum number of equations necessary to reproduce experimentally measured tissue characteristics. SQP constrained optimization is applied to optimize the model parameters to reproduce the physiology of human heart cells. Pseudospectral methods are proved to have higher order of convergence in the numerical solution of the reaction-diffusion partial differential equations arising in the propagation of electrical activity in the heart. Since heart anatomy is highly irregular, we propose novel techniques to extend pseudospectral methods to irregular domains [2],[3]. Applications of the present methodology to the study of genetic diseases linked to sudden death [4] are discussed.

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Level set models for blind deconvolution of baroque paintings

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From a mathematical point of view, paintings are a special class of images, in the sense that they have features which may be used in order to improve their analysis. On the other side, paintings are spoiled by some factors (weather, time, ...) which restrict the general processing problems. One of the most important processes in the scientific stages of artistic restoration, is deconvolution (or deblurring of the painting).

Usually, blind deconvolution can be defined as the problem of recovering an original image, u_0, from a deteriorated one, u, according to the relationship:

$$u = k * u_0 + n$$

where the convolution kernel, k, is unknown, and n is some noise present in the actual image, u.

The problem above is ill posed, and, in order to be able to get a solution, we need to settle some information about u_0, which relates to some restrictions in a given norm.

In this work, we will use knowledge about the paintings to choose the norm, and to settle the restrictions both on k and on u_0, in order to obtain a model for restoration of paintings. We will apply it to some pieces of Spanish Baroque which, in the moment of this research, were being restored. We will compare our computational results with those of the professional, and artistic, restorers.

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Past expectations and present prices – hysteresis in a simple economy

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Economic agents are defined by their resources, preferences and expectations. In the marketplace, they are mediated by a price system, which defines the terms of trade. If demand for a good exceeds supply, the price rises; if there is an excess of supply, the price lowers. This tatônnement process can end up in an equilibrium point. When there is more than one equilibrium point, path-dependence becomes an issue (see Arrow and Hurwicz [1] and Arrow *et al* [2]).

We use bifurcation theory on a simple model and find robust hysteresic behaviour in the economy: a temporary change in the parameters may have a permanent effect on the economic outcome.

Our analysis extends Bala's [3] study of pitchfork bifurcations in the tatônnement process. We use unfolding theory and a weighted homogeneous normal form to show that economically meaningful perturbations provide a universal unfolding for the pitchfork (Golubitsky and Schaeffer [4]). This allows us to describe qualitatively the price dynamics in the neighborhood of the bifurcation, and to prove the existence of hysteresis.

We illustrate hysteresis in a numerical example with two agents. A temporary perturbation of expectations influences, in a permanent way, equilibrium selection, that is, the economic outcome in terms of prices and final consumption.

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Propagation of edge wave in an initially stressed anisotropic medium

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This paper investigates the propagation of edge waves in an initially stressed anisotropic plate of finite thickness and infinite length. The velocity equation of edge wave in an anisotropic initially stressed plate has been derived and the results of numerical calculations are presented graphically. The velocity has been computed for various initial stress parameters and different anisotropy ratios. Some particular cases have been discussed to get the velocity in an initially stress free and anisotropic medium. The velocity of Rayleigh wave has been obtained in a half space as a particular case. It was found that the phase velocity of edge waves is considerably influenced by prestressing present in the medium. The study reveals that the presence of compressive initial stress first increase the velocity of edge waves up to a certain value and then decreases to its minimum value and for tensile initial stresses, velocity of propagation first increases rapidly and then decreases. The velocity drops down for large initial stresses. This is due to the fact that surface instability of the medium starts.

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A methodology to design control laws for computational aeroelasticity

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The purpose of this work is to develop a methodology for designing active control laws in a computational aeroelasticity environment.

The methodology involves employing a systems identification technique to develop an explicit state-space model for control law design from the output of a computational aeroelasticity code. The particular computational aeroelasticity code employed in this paper solves the transonic small disturbance equation using a time-accurate, finite-difference scheme.

Linear structural dynamics equations are integrated simultaneously with the computational fluid dynamics equations to determine the time responses of the structural outputs. These structural outputs are employed as the input to a modern systems identification technique that determines the Markov parameters of an "equivalent linear system".

The eigensystem realization algorithm is then employed to develop an explicit state-space model of the equivalent linear system. Although there are many control law design techniques available, the standard Linear Quadratic Gaussian technique is employed in this paper. The resulting control law is of the form:

$\{\dot{\hat{x}}\} = [\bar{A}]\{\hat{x}\} + [L]\{y\}; \{u\} = [-G]\{\hat{x}\}$, where $\hat{x}$ is the estimate of the state vector x, G is the state feedback gain matrix and L is the Kalman filter gain matrix.

The computational aeroelasticity code is modified to accept control laws and perform closed-loop simulations. Flutter control of a rectangular wing model is chosen to demonstrate the methodology.

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Numerical simulation of large ice masses evolution

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This work deals with the numerical solution of a complex mathematical model arising in theoretical glaciology. The idea of an ice sheet is a large highly viscous ice drop, where the steady state equilibrium can be briefly explained as a consequence of the balance between the ice accumulation taking place at the top and the ablation processes that mainly occur at the margins. So, the global moving boundary problem governs thermomechanical processes jointly with ice sheet hydrodynamics. A simplified model can be framed into the *shallow ice approximation* [4], which takes into account the ice mass length and width scales to neglect some terms in the original conservation equations issued from continuum mechanics. Thus, the proposed highly nonlinear system of pde's governs three main problems: the upper profile evolution, the ice velocity field and the temperature distribution. Each problem requires the solution of the other two ones, so that a fixed point iteration between them seems a possible numerical technique [2]. Indeed, the profile and temperature models are posed as free boundary problems whereby the ice sheet extent and the interface between cold and temperate ice are additional unknowns. Moreover, to obtain real atmospheric boundary conditions it is necessary to take into account an environmental energy equation [3]. Concerning to the numerical methods, for the profile problem a Lagrange–Galerkin approximation is combined with a duality algorithm for maximal monotone operators [1]. In the thermal problem, besides the appropriate upwinding time scheme and piecewise linear Lagrange finite elements in space discretizations, nonlinear viscous terms are treated by a Newton method, and two phase Stefan formulation and Signorini boundary condition by duality methods. The computation of velocity field inside the ice is performed by numerical quadrature. A simulation example involving real data issued from Antarctic shows the temperature, profile and velocity qualitative behaviour as well as the free boundaries and basal effects by means of an original and specific numerical simulation toolbox.

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Analysis of an age-dependent SI epidemic model with disease-induced mortality and proportionate mixing assumption: the case of vertically transmitted diseases

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An SI epidemic model for a vertically as well as horizontally transmitted disease is investigated when the fertility, natural mortality and disease-induced mortality rates depend on age and the force of infection corresponds to a special form of intercohort transmission called proportionate mixing. We determine the steady states and obtain explicitly computable threshold conditions, and then study the stability of the steady states.

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The effect of anomalous diffusion in chemotactic aggregation

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One of the most studied models for chemotactic aggregation is the Keller-Segel model, a system of two parabolic partial differential equations. One of these equations represents the evolution in time of the cellular population, that is driven by diffusion and a chemotactic drift. The other represents the evolution of the chemical, that is also supposed to diffuse and to be segregated by the cells. This model has blowing up solutions for dimensions greater or equal than two, a mathematical fact that crucially affects the patterns that can form in the biological system. In fact, this implies that in a three-dimensional system, while collapse to infinite density lines and points can occur, collapse to an infinite density sheet is mathematically impossible. Correspondingly, in a two dimensional system, it is impossible to find collapse to an infinite density line, but it is still possible to observe collapse to an infinite density point [1]. This successfully explained experiments performed with *Escherichia coli* [2, 3], and constituted a remarkable achievement of mathematical biology.

However, the Keller-Segel model has some limitations in its applicability. One of the strongest assumptions of this model is the diffusive character of the cellular motion, known to be false in many situations. In particular, mesenchymal cells perform a sort of nonlocal random walk that is better modelled by an integro-differential operator [4]. With this modified diffusion, we proved that the new partial differential system has blowing up solutions in one dimension [4], solutions that explain the patterns observed in a recent *in vitro* experiment [5], and that were absent in the case of ordinary diffusion. Motivated by this result, we modified the Keller-Segel system to model a cellular population performing Lévy flights, instead of the usual random walks, as found in several experiments. For this case, we provide global existence results in one dimension.

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New method in quantum information theory

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Recent progress in experimental realisation of quantum computations raised new questions in quantum information theory [1]. Among them we mention quantum state transfer from one end of the chain to another (spin chain $s = 1/2$ is a model of the quantum register) [2], and qubit addressing problem based on one-dimensional spin systems with different Larmor frequencies [3, 4].

We develop a new method of diagonalisation of the XY-Hamiltonian of inhomogeneous open linear spin chains with periodic (in space) changing Larmor frequencies and coupling constants [4]. For chains with $kn-1$ sites (spins) and period k we obtain an explicit diagonalisation of the corresponding XY-Hamiltonian.

The method is applied to different problems of multiple-spin dynamics. In particular, we calculate the distribution of intensities of multiple-quantum (MQ) coherences for MQ NMR spectroscopy in solids [4].

We propose a future application of the method in analysis of experiments modeling quantum computations based on nuclear magnetic resonance (NMR).

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Subsidies in an international differentiated duopoly with unknown costs

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We consider two Cournot firms, one located in the home country and the other in the foreign country, producing differentiated goods for consumption in a third country or market. We suppose that the home firm has two different technologies, and chooses one of them following a binary probability distribution. The utilization of one or the other technology affects the unitary production cost. At the beginning of period 1, the home government commits to a subsidy for the home firm. At this stage neither the home government nor the foreign firm knows the cost of the home firm, though it is common knowledge that it has either low (c_L) or high (c_H) costs with $\mathrm{Prob}(c = c_L) = \phi$. Marginal cost of the foreign firm is common knowledge. At the end of the first period, both firms make their output decisions simultaneously, to maximise profits. After observing the output levels of the home firm, the uninformed agents update their beliefs about the costs of the home firm. Let $\phi(q_1)$ be the common updated probability assessment, where q_1 is the first-period output of the home firm. At the beginning of the second period, based on the updated beliefs, $\phi(q_1)$, the home government sets its policy instrument level to maximise welfare. Given the subsidy level chosen by the home government and given the updated beliefs of the foreign firm, the two firms choose period 2 outputs to maximise profit.

D. Wright [2] studied the case of homogeneous goods, and showed that, in such case, the optimal subsidy is lower when the home firm signals costs compared to the case when the firm does not. In our work, we show that if the goods are differentiated, the optimal subsidy can be higher when the home firm signals costs compared to the case when the firm does not.

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Leadership disadvantages on a differentiated model when demand is uncertainty

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Von Stackelberg [2] proposed a duopoly model where the sequential nature of choices is an alternative to the simultaneity of moves in Cournot model or in Bertrand model. In the Stackelberg model, the leading firm chooses the output level first, taking into account the follower's optimal response to its quantity choice. Then, the follower sets its output level based on the leader's choice. In the case of complete information, the typical situation is that the position of leader is most preferred and that the follower's position is least desirable. However, in a case of incomplete information, we show that the first mover does not necessarily have advantage over the second one.

We consider a Stackelberg model in which the firms produce a differentiated good, and with demand uncertainty only for the first mover. Let $\beta \geq 1$ represent the degree of substitutability of the products, and let the demand intercept, α, be a random variable uniformly distributed in the interval $[\underline{\alpha}, \overline{\alpha}]$, with $\overline{\alpha} > \underline{\alpha} > 0$. In our model, the first mover chooses its output level according to the expected demand, while the second mover chooses its output level knowing the exact realised demand. We prove that there are constants $C_{1,\beta}$ and $C_{2,\beta}$, that depend only upon β, such that (i) if $\overline{\alpha}/\underline{\alpha} < C_{1,\beta}$, then we get that the leading firm profits more than the follower; (ii) if $C_{1,\beta} \leq \overline{\alpha}/\underline{\alpha} \leq C_{2,\beta}$, then we get that when the realised demand is very low, the leading firm profits less than the follower, otherwise the leading firm profits more than the follower; and (iii) if $\overline{\alpha}/\underline{\alpha} > C_{2,\beta}$, then we get that when the realised demand is very low or very high, the leading firm profits less than the follower, and when the realised demand is in an intermediate region, the leading firm profits more than the follower. Liu [1] showed a similar result in the simplest case of homogeneous products. In the case of homogeneous goods the constants $C_{1,\beta}$ and $C_{2,\beta}$ are equal. So, in our model, it appears one more special situation, the above item (ii).

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Prisoner's Dilemma in an Edgeworthian Economy

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We present a model of an Edgeworthian exchange economy where two goods are traded in a market place. The novelty of our model is that we associate a greediness factor to each participant which brings up a game alike the prisoner's dilemma into the usual Edgeworth exchange economy. Along the time, random pairs of participants are chosen, and they trade or not according to their greediness. If the two participants trade then their new allocations are in the core determined by their Cobb-Douglas utility functions. The exact location in the core is decided by their greediness with an advantage to the greedier participant. However, if both participants are too greedy, they are penalized by not trading. We analyze the effect of the greediness factors in the variations of the individual amount of goods and in the increase of the value of their utilities. We show that it is better to be in minority. For instance, if there are more greedy participants, the increase of the value of their utilities is smaller than the increase of the value of the utilities of the non greedy participants.

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Optimal operating and scraping policies for equipment

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In [1, 2] we examined the problem of optimal service life of equipment in the framework of the multi-period replacement model without imposing equidistant replacement periods as in [3]. In the present work we consider a single operating period for which we examine the relationship between the operating policies of utilization-maintenance: and the scraping policy of operating duration: T. For this purpose we cast it as the following problem in the context of Optimal Control Theory:

$$\max_{u,m,T} \left\{ A = \tilde{Q} + \tilde{S} = \int_0^T e^{-\sigma t} r(u,m) K^{\varepsilon} e^{\zeta t} dt + e^{-\sigma T} p e^{\eta T} K \right\}$$
$$with \;\; \dot{K} = -w(u,m)K, \;\; K(0) = K_0$$

We distinguish two types of capital, the operating revenue capital: K^{ε}, and the scrap revenue capital: K, and we note that they are affected by the operations and by time discounting at different rates. Using this distinction we find the following:
1. Concerning profitability, it depends on the relation between the discount rate of the scrapping capital $\sigma - \eta$ and the price of the equipment.
2. Concerning scraping policies, "high-priced" equipment is scrapable:$T < \infty$ iff on the average the operating capital deteriorates faster then the scraping capital: $\varepsilon w - \zeta > w - \eta$, otherwise it is durable: $T = \infty$. The opposite happens if the equipment is "low-priced"
3. Concerning operating policies, in time they move in time from “harder” to “softer” if the operating capital is discounted more heavily than the scraping capital: $(\sigma - \zeta)/\varepsilon > \sigma - \eta$, conversely in the opposite case.

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A theory of thermoelastic continua with big voids

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We present here a model of a continuum with ellipsoidal microstructure for the study of a thermoelastic material with large pores, in presence of discontinuity surfaces, and propose the appropriate constitutive relations along with the thermodynamic restrictions and the invariance principles (see [1] for the general theory of continua with microstructure).

We also define particular weak singularities, called macro–acceleration waves, for which only jumps of the derivatives of the macro– and micro–displacement of order 2 and of the temperature of order 1 are of interest in the theory (see, also, [4] and [3]). We study these (homothermal) waves for a linear conducting homogeneous centrosymmetric isotropic material [2] and derive the propagation conditions and the growth equations governing the motion of them in order to discuss the eventual couplings between macro– and micro–waves.

In general, for the homothermal macro–acceleration waves, three speeds of propagation are possible: the one related to a shear-optical micro-wave completely decoupled from the macro–mechanical thermoelastic properties; the second related to a transverse micro-wave coupled with a transverse macro–acceleration wave, spreading without perturbing the thermal field; the third related to an extensional micro-wave coupled with a longitudinal macro–wave and with a discontinuity in the thermal field.

Finally, we note that the waves of compaction or distention, usually predicted in voids theories [5], cannot occur in the general theory, unless we impose additional conditions: in this case, the two micro–waves are coupled both with thermal discontinuities and with longitudinal mechanical macro–waves.

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The semigroup of film casting: linear transport and elliptic constraints

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The mathematical description of free liquid films formed by film casting poses serious analytical challenges. In the simplest case of a highly viscous material, this description, essentially due to Y. L. Yeow [4], takes the form of a nonlinear transport equation coupled to an elliptic system of momentum equations in two-dimensional space.

In this presentation we will report about recent results in [1, 3]. We will concentrate on important issues related to the spectral and linear stability of stationary solutions in film casting. Specifically, we will show that the linearized equations satisfy the semigroup property and that the semigroup, being non-analytic, gains in smoothness as time progresses. This latter result is based on elliptic estimates for the system of linear momentum equations that do not follow from standard elliptic theory. Related one-dimensional flows have been studied before [2].

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Numerical simulations of the attack of an opportunist virus to the infected immune system by the HIV

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It is important, when trying to describe biological phenomena, to project real situations into computer simulations. This allows us to predict and experiment with the computer a number of real life situations. We take this approach in our study of the HIV infection.

Martin Nowak and Charles Bangham [1, 2] introduced some HIV propagation models in the human body. Building on their work, we developed an integro-differential model that simulates the HIV in immune system. This model preserves the good properties Nowak-Bangham one, but it is also capable of simulating the mutation phenomena, except for the last stage of the virus, when it breaks the immune system down. More recently, we developed a more soffisticated model that deals with the complete process including the last stage through numerical simulations.

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Spiral instabilities in Rayleigh-Bénard convection under localized heating

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We study, from the numerical point of view, instabilities developed in a fluid layer with a free surface, in a cylindrical container which is non-homogeneously heated from below. In particular we consider the case in which the applied heat is localized around the origin approaching a boundary condition for a thermal plume. The numerical method is a Chebyshev collocation method in the primitive variables formulation [1, 2]. The localized boundary condition introduces different scales in the problem. The convergence of the method in its current form is restricted to some ranges of the applied temperature pulse. An axysimmetric basic state appears as soon a non-zero lateral temperature gradient is imposed. The basic state may bifurcate to different solutions depending on vertical and lateral temperature gradients and on the shape of the heating. Four regions with different solutions and bifurcations can be distinguished depending on those parameters: giant spirals and targets, oscillatory and stationary bifurcations with medium wave numbers, stationary bifurcations with large wave numbers and the absence of bifurcation (at least in the studied parameter set). Localized structures both at the origin and at the outer part of the cylinder may appear either as Hopf or stationary bifurcations. The influence of other parameters as aspect ratio and Prandtl number is also studied [2].

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Hopf bifurcation and structural instability in an open economy with Keynesian rigidity

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This paper attempts to contribute to the debate in macroeconomic dynamics by presenting the neoKeynesian challenge. Proof is presented regarding the behavior of an open-economy two-sector growth model in the neoKeyenesian tradition of non-market clearing. It has been shown that there possibly exists a Hopf-bifurcation type of structural instability in a nonlinear dynamical model of the macroeconomy by which a stable region is connected to an unstable region situated in a center manifold in the state space of the resulting dynamical system. The Keynesian view that structural instability globally exists in the aggregate economy is put forward, and therefore the need arises for policy to alleviate this instability in the form of dampened fluctuations is presented as an alternative view for macroeconomic theorizing.

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Greens function approach in water wave scattering problems

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In the recent past, there is a significant progress in the literature on the interaction of surface waves with large floating structures. A great deal of effort has been made in studying, designing and building very large floating structures (VLFS) for utilization of ocean space, the branch of hydroelasticity has gained considerable importance in the present scenario. Before the construction and positioning of any VLFS, careful and detailed studies are needed to investigate the hydrodynamic and hydroelastic behavior of the system. To overcome the computational difficulty, the structure is often assumed to be semi-infinitely long in comparison with the wavelength of the incident wave. The hydroelastic deformations are considered in case the body is itself flexible or the body is very thin compared to wave parameters. The oblique wave scattering by an articulated floating elastic plate in water of infinite depth is analyzed in the linearized theory of water waves. Using the geometrical symmetry of the articulated plate, the associated boundary value problem in the half plane is reduced to two boundary value problems in the quarter plane, whose solutions are derived by the direct application of Green's function approach. The articulated plate is modelled as the assembling of two semi-infinite thin elastic plates attached by connectors. The hydroelastic behavior of the floating elastic plate can be investigated by analyzing the stiffness of the connectors on the reflection and transmission characteristics of the flexural gravity waves. The reflection and transmission coefficient can be easily computed and analyzed to understand the effect of articulation on the wave motion below the plate. The present method is elegant and simpler than the existing method of solution to analyze similar problems and can be easily applied to a large class of problems in the area of wave structure interaction in the field of Ocean Engineering and other fluid structure interaction problems arising in various branches of engineering and mathematical physics.

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Sampling-reconstruction procedure of Markov processes with jumps

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The Sampling - Reconstruction Procedure (SRP) of the Markov processes *with jumps* is not investigated in the literature. We suggest to investigate this problem on the basis of the conditional mean rule [1]. This rule has been applied in the statistical description of the SRP of some Gaussian and non Gaussian processes [2] and [3]. We discuss some specific features in order to apply this rule in the SRP description of processes with jumps. The main problem is to determine the estimation of the jump point from the given state to another when two different samples are known. This estimation point is the reconstruction point and the variance of this estimation is the reconstruction error. We propose to determine the sampling interval taking into account two circumstances: the given estimation error and the given probability of an undetected jump. The general solution is obtained. As a particular case the SRP of the Binary Markov Process is considered. There is the main result here: the conditional probability function of the jump point between two different samples is the cut exponential function. The analytical expressions for the jump point estimation and for the variance of the estimation are given. The SRP example with three states of the Markov jump process is considered also. The results of the statistical simulation for both examples are presented. There is a good coincidence between the theoretical and simulation results.

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Stability and optimal harvesting in a stage structure predator-prey switching model

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This paper is concerened with a stage structure predator-prey interaction where the prey is a stage structure with two life stages immature and mature. The predator consumes both the young and adult of the prey and the prey population is more prone to predator at higher densities. Local and global stabilities of the equilibrium sets are discussed. With harvesting for the mature population we obtain conditions for a threshold of the harvesting for sustainable yield.

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On stability of the interface between two fluids: undercompressive shocks, flow reversal, and compressibility effects

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The first part of this presentation discusses the flow of two incompressible immiscible viscous fluids in a channel. Using long-wave (lubrication) approximation, and the scaling appropriate to liquid/gas flows, we derive a nonlinear evolution equation that governs the interface separating the two fluids and the leading-order pressure, including the effects of viscosity stratification, inertia, shear, and capillarity [1, 2]. In particular, we consider the case resulting in a single evolution equation whose dynamics depends nonlocally on the interfacial shape. We find admissible criteria for existence of Lax shocks, undercompressive shocks and rarefaction waves, which have been also recently reported in different context [3].

The second part concentrates on the compressibility effects in two fluid channel flows. Assuming ideal gas behavior and isothermal conditions, our approach leads to a system of high order nonlinear partial differential equations describing the evolution of the interface, and of the temporal and spatial density distribution of the gas [4]. A linear stability analysis requires solving numerically a boundary-value problem for the gas density and interfacial deviations from the base state. We find that the gas compressibility has destabilizing effect on the interfacial stability in the limit of vanishingly small wavenumber. However, for finite wavenumbers, compressibility may have stabilizing effects, so that sufficient shear is required to destabilize the flow. We discuss the consequences of these new results on the two-phase flows in applications including oil and gas flows, and flows in microchannels.

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A model for vertical and oral transmission of Trypanosoma cruzi

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Recent research in the transmission of the protozoan parasite *Trypanosoma cruzi*, some strains of which cause Chagas' disease, suggests that the traditional biting/fecal infection route from vector to host may not be responsible for maintaining the observed prevalence of some strains among sylvatic hosts such as raccoons [4, 5]. Rather, consumption of infected vectors by hosts may establish the infection [1], and vertical transmission among placental hosts may sustain it. This hypothesis is supported by the higher prevalence among placental hosts than marsupials. A system of ordinary differential equations provides a model with which to evaluate the relative importance of these transmission routes and show, through the infection's basic reproductive number, how the combination of oral and vertical transmission can sustain an endemic state [3]. Qualitative analysis techniques from dynamical systems theory show the effects of superimposing this predator-prey structure on the host-vector infection model, including the existence of multiple attractors [2].

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Heat transfer of carbon nanotubes: A Lie symmetry approach

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A one-dimensional model for heat transfer in carbon-nanotubes is considered [5]. The Lie group theory is used to implement the symmetry principle on the model. The symmetry principle is employed to obtain the form of the arbitrary function in the model, this is the essence of the group classification method [1, 2, 3, 4]. Many models in real-life applications contain parameters or functions which cannot be determined from any known physical law, hence a need to use the method of group classification to specify their forms.

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Computing eigenvalues of a Chebyshev collocation approximation to a thermoconvective instability

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The aim of our work is to develop efficient numerical methods for computing eigenvalues for the stability analysis of a problem involving the motion of a fluid within a cylindrical container heated non-homogeneously at the bottom [3]. The partial differential equations that model this problem are discretized with a Chebyshev collocation method with appropriate conditions for the pressure field [1]. So the steady states are numerically calculated. Its linear stability is formulated with a generalized eigenvalue problem. This eigenvalue problem presents an original block matrix structure where one of the submatrices is singular. The numerical approach (generalized Arnoldi method) utilizes the idea of preconditioning the eigenvalue problem with a modified Cayley transformation before applying Arnoldi method [2, 4]. This method allows affective computation of the critical eigenvalues which determine wether the steady flow is stable or unstable and to calculate the bifurcation points. Both types of bifurcations, stationary and oscillatory are detected. A comparison in computing time between this method and the QZ method shows the computation is more efficient with the generalized Arnoldi method. Via pseudospectra calculations the reliability of the eigenvalue calculations and bifurcations is proved.

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A model of plant competition for sunlight

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We examine a mathematical model of competition between two plant species with clonal growth form that interact only by their shared use of sunlight [1, 2]. The model takes the form of a Kolmogorov-type system of nonlocal integro-differential equations in which the specific growth rate function of each species is a functional of both species' fixed vertical leaf profiles [3]. We use implicit methods and nullcine endpoint analysis to show that, under certain conditions, the species' nullclines can intersect at most once, and that when they do intersect, coexistence is always stable. We also partition parameter space into regions within which either competitive exclusion or competitive coexistence occurs. We conclude that canopy partitioning is both necessary and, under appropriate parameter values, sufficient for the stable coexistence of our two hypothetical plant species.

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The population dynamics of the malaria vector

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A deterministic differential equation model for the population dynamics of the human malaria vector is derived and studied. Conditions for the existence and stability of a non-zero steady state vector population density are derived. These reveal that a threshold parameter, the vectorial basic reproduction number, exist and the vector can established itself in the community if and only if this parameter exceeds unity. When a non-zero steady state population density exists, it can be stable but it can also be driven to instability via a Hopf Bifurcation to periodic solutions, as a parameter is varied in parameter space. By considering a special case, an asymptotic perturbation analysis is used to derive the amplitude of the oscillating solutions for the full non-linear system. The present modelling exercise and results show that it is possible to study the population dynamics of disease vectors, and hence oscillatory behaviour as it is often observed in most indirectly transmitted infectious diseases of humans, without recourse to external seasonal forcing.

In this communication, we derive and study a simple model for the dynamics of the human malaria vector based on the simple idea that the mosquito has a human biting habit. Since it is the mosquito that actively seeks and bites human beings, this assumption, which has been used successfully to model the dynamics of malaria transmission Ngwa et al. [1, 2, 3], may be seen as a restricted form of homogeneous mixing based on the idea that the mosquito has a human biting habit. The concept of the existence of a basic reproduction number, R_0, in models has been addressed by Porphyre et al. [4], Diekmann et. al. [5]. In our formulation, R_0 depends on a mass action contact rate τ^* as well as on the probability of the mosquito obtaining a successful blood meal p in the sense that $R_0 \to 0$ when ever $p \to 0$ or $\tau^* \to 0$ and saturates to a positive non-zero value when $p \to 1$ and $\tau^* \to \infty$.

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Mathematical modelling of radiotherapy-implications for advanced head and neck and prostate cancer

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The standard Linear-Quadratic (LQ) model for radiotherapy is used to investigate different schedules of radiation treatment planning, to study how these may be affected by different tumour repopulation kinetics between treatments. The laws for tumour cell repopulation include the logistic and Gompertz models and this extends the work of Wheldon et al [1],where the case of exponetial re-growth between treatments was studied. Additionally we consider the restricted exponential model. This has been successfully used by Panetta and Adam in the case of chemotherapy treament planning. Treament schedules investigated include standard fractionation of daily treatments, weekday treatments, accelerated fractionation, optimised uniform schedules and variation of the dosage and $\frac{\alpha}{\beta}$ ratio. The various tumour cell population growth models are applied to treatment schedules for advanced head and neck cancer [3]. Prostate cancer is also considered [4]. Calculations based on our analysis indicate that even with the tumour cell population laws scaled, to mimic initial growth, so that growth mechanisms are comparable, variations in the survival fraction of orders of magnitude are seen to emerge. The logistic and exponetial models produce similiar results but the Gompertz model is significantly less effective in eradicating tumour cell populations. The present study also indicates that the faster the rate of growth of the tumour and the higher the repair capacity of the cell line, the greater the variation in outcome of the survival fraction. Gaps in treatment, planned or unplanned, are found to accentuate the differences of the survival fraction given alternative growth dynamics.

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Solution of $\mathbb{R}$-linear conjugation problems modelling power fields distribution in regular composites

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Complex variable methods are applied to study some double-periodic, multiphase (two–four phase), planar heterogeneous structures.

Namely, it is considered a two-dimensional, stationary field $\mathbf{v}(x,y) = (v_x, v_y) = \mathbf{v}_p(x,y)$, $(x,y) \in \Omega_p$, which is potential and solenoidal in each isotropic component Ω_p of a heterogeneous medium:

$$\operatorname{div} \mathbf{v}_p(x,y) = 0, \qquad \operatorname{curl} \mathbf{v}_p(x,y) = 0, \tag{1}$$

and along an interface L_{pq} between dissimilar phases Ω_p and Ω_q normal components of vectors $\mathbf{v}_p$ and $\mathbf{v}_q$ are equal and their tangential components are proportional:

$$\mathbf{v}_{pn}(x,y) = \mathbf{v}_{qn}(x,y), \qquad \rho_p \mathbf{v}_{p\tau}(x,y) = \rho_q \mathbf{v}_{q\tau}(x,y),\ (x,y) \in L_{pq}. \tag{2}$$

where $\rho_p \geq 0$ ($\sigma_p = 1/\rho_p$) is the coefficient of resistivity (conductivity) of the phase Ω_p. In terms of complex variable, $z = x + iy$, the boundary value problem (2) is equivalent to the following problem of $\mathbb{R}$-linear conjugation:

$$v_p(t) = A_{pq}\, v_q(t) - (1 - A_{pq})\, [t'(s)]^{-2}\, \overline{v_q(t)}, \quad t \in L_{pq}, \tag{3}$$

where, due to (1), $v(z) = v_x(z) - i\, v_y(z)$ is a piecewise–holomorphic function and $A_{pq} = (\rho_p + \rho_q)/2\rho_p$.

The problem (3), generally unsolvable analytically, is solved in closed form for some specific double-periodic structures (two-four phase rectangular, regular triangular ([1], [2]) and rhombic ([3]) checkerboard structures). Explicit analytical expressions of functionals of energy dissipation and of effective resistivities (conductivities) are found as one of possible applications of the solutions derived. The latter results generalize, in particular, the well-known Keller-Dyxne-Mendelson formula [4] and prove the long-standing Mortola and Steffé conjecture [5].

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Applications of the Loewner equation to crack propagation in brittle solids

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A. A. Griffith in [1] laid the foundations of the modern approach to brittle fracture. He considered the rupture problem as a competition between two different energies involved during crack growth: The stored elastic energy released (reversible) and the crack surface energy (dissipative). He was able to write one scalar equation showing the condition of critical equilibrium for a crack configuration. In a two dimensional setting at least two scalar relationships are needed to determine the subsequent crack path.

In this work we study the propagation of a crack in critical equilibrium for a brittle material in a Mode III field. The energy variations for small virtual extensions of the crack are handled in a novel way: the amount of energy released is written as a functional over a compact family of univalent functions on the upper half plane. Classical techniques developed in connection to the Bieberbach Conjecture are used to quantify the energy-shape relationship. We apply Schiffer's boundary variation technique (see [2]) to find optimal paths in the sense of maximum elastic energy released. By means of a suitable parameterization generated by the Löwner equation we impose a stability condition on the field which derives in a local crack propagation criterion. We called this the *anti-symmetry* principle, being closely related to the well known symmetry principle for the in-plane fields.

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Local immune responses adjusted by regulatory T cells

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A mathematical model for inhibition of interleukine 2 (IL-2) secretion due to Regulatory T cells (Treg) is presented. This model has a quorum T cell population threshold that needs to be overtaken in order to assure an immune response and a lower threshold to reach again the controlled state. The inhibition of IL-2 secretion by Tregs increases both thresholds. This shift can be controlled locally for different tissues by adjusting the local Treg population size. Cross reactivity to pathogens and bystander proliferation on unrelated immune responses can overcome an initially controlled state, being the trigger to an autoimmune response by T cells.

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On the transverse motions under heavy loads of thin beams with variable prestress

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In this paper, the effect of variable axial force on the dynamic response of elastic beam resting on elastic foundation and subjected to concentrated loads is investigated. The fourth order partial differential equation with variable and singular coefficients governing the motion of the elastic beam is solved using Generalized Galerkin's Method and the Modified asymptotic method of Struble. It is established by both Analytical and Numerical Analysis that, the higher the values of the axial force N and the foundation rigidity K, the lower the response amplitudes of the elastic thin beam with variable prestress when it is under the action of concentrated moving loads. Furthermore, it is found that the critical velocity for the system under the influence of the moving force is greater than that under the influence of the moving mass first approximation and moving mass entire beam model. Hence resonance is reached earlier in the latter.

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On the efficient frontier associated to portfolio selection models

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In the present paper we associate the notions of efficient frontier set and efficient frontier map to a parametric optimization problem. Five portfolio selection models are defined: two minimum risk models, two maximum expected return models and a compromise optimization model. In [1] - p.8 R. Korn proved for the mean-variance portfolio selection models the following result:

Theorem: If the covariance matrix of the random vector of returns is nonsingular then the efficient frontier sets associated to the minimum risk models and the maximum expected return models are equal.

We give a counterexample showing that R. Korn's result does not hold for an arbitrary vector of expected returns. Our result states that Korn's theorem holds if and only if there is only one asset which has the greatest expected return. Taking into account various hypotheses on the covariance matrix and on the vector of expected returns are proved several relations between the efficient frontier sets and the efficient frontier maps associated to the mean-variance portfolio selection models.

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Spatial synchronization and extinction of species under external forcing

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Using a general model we show that under a common external forcing, the species with a quadratic saturation term in the population dynamics first undergoes spatial synchronization and then extinction, thereby avoiding the rescue effect. This is because the saturation term reduces the synchronization time scale but not the extinction time scale. The effect can be observed even when the external forcing acts only on some locations provided there is a synchronizing term in the dynamics. Absence of the quadratic saturation term can help the species to avoid extinction.

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Chemical elements, a topological space?

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We describe a mathematical methodology to provide with a topology a finite set Q of chemical interest [1, 2, 3]. Procedure begins searching for similarities among the $q \in Q$. To do this we use cluster analysis (CA), a mathematical methodology that uses similarity functions (some of them metrics) and grouping methodologies (set point distances). The final result of CA is a dendrogram D, which might be considered as a two-dimensional plot of similarities among $q \in Q$ [1, 2, 3, 4, 5]. The second step of our methodology is the extraction of the neighborhoods shown by the dendrogram D. To do this we developed a mathematical procedure considering a dendrogram as a graph [1, 3]. We developed a theorem showing that each branch $B \in D$ is an open set of the basis $\mathfrak{B}_n$ for the topology τ_n. Having defined $\mathfrak{B}_n$, we calculate several topological properties for several $M \subset Q$ of chemical interest [1, 2, 3]. The topological properties studied are closures $\bar{M}$, derived sets M', boundaries $b(M)$, interiors $Int(M)$ and exteriors $Ext(M)$. Some of the results of this procedure [4] are, among others, that the boundary of the set of metals and non-metals is the same set [1, 3, 5], the set of semimetals. Another result is that the set of alkali metals and noble gases are perfect sets according to their topological properties.

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A quasistatic contact problem for viscoelastic materials with long memory involving damage

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Phenomena of contact between deformable bodies or between deformable and rigid bodies abound in industry and everyday life. Because of the importance of contact processes in structural and mechanical systems, considerable effort has been put into modeling, analysis and numerical simulations, and the literature in this field is extensive. The effective functioning and safety of a mechanical system may be deteriorated as the material undergoes damaged due to the opening and growth of microcracks which lead to the decrease in the load carrying capacity of the body. Because of the importance of this topic, an increasing number of publications dealing with damage models appeared in the last decade. Mathematical problems including damage have been studied recently for elastic, elastic-viscoplastic, or viscoelastic materials (see, e.g. [1, 2]).

Here, a contact problem between a viscoelastic body with long memory and a deformable obstacle is studied. The evolution of damage is modeled following the ideas in the early works in [3, 4] and the memory is modeled by a non-linear generalization of the model used in, for example, [5]. The variational analysis of this problem leads to the study of a system involving an elliptic evolutionary variational equation derived from the principle of virtual works and a parabolic ordinary differential equation modeling the evolution of damage. Recent results involving the existence of a unique solution to the system and the numerical analysis of a fully discrete problem will be presented. Finally, numerical simulations in the study of two-dimensional test problems will be shown.

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Mathematical inverse problem for computing seismic wave speeds. Part 1: vertical sensors; Part 2: horizontal sensors

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A useful procedure to study ground characteristics is by measuring seismic wave propagation speed. To achieve this a pulse of energy (small explosion) is released near the surface of the ground. A number of sensors are distributed at different positions in or on the ground. The travel times for the passage of the pulse from the explosion point to the different sensors are recorded. This work is to show how to compute wave speed profiles as a mathematical inverse problem. Two cases are considered: i) horizontally, and ii) vertically distributed sensors. For each case, the solution of the direct problem is achieved using variational methods. Numerical simulation is used to create input data (arrival times) for the inverse problem. Then, use is made of optimization techniques to solve the inverse problem. Excellent results are obtained for all the various examples considered.

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Asymptotic stability of nonlinear delay malaria epidemic model

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Malaria is a very old disease originating in Africa, spreading as humankind migrated to other lands. The disease gets its name from Italian word for "bad air". The death of many infants would be expected during a malaria epidemic, partially because flaciparum induces high rates of miscarriages and infant death. The mosquitoes that transmit malaria flourish in marshy areas. In fact, malaria kills more than 2 million people each year-that is about 700/day. More than one million of these are children under the age 5. In addition to the millions who die, up to half billion suffer the effects of malaria. Because mothers are more likely to suffer malarial relapses during pregnancy malaria is an important cause of low-weight births and still births. More than half of miscarriages in endemic areas are caused by malaria. In this paper, we investigate the asymptotic stability of the nonlinear delay mathematical model of malaria based on the relation between humans and mosquitoes. The model consists of two nonlinear delay differential equations with two different delays. Our aim is to establish some sufficient conditions for the local asymptotic stability by analyzing the corresponding chractristic equations and by aplying the Lyapunov functional method. The conditions ensure that the disease always locally endemic.

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Reconstructing convergent G^1 B-spline surfaces for adapting the quad partition

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In [1], we provided a local scheme of constructing convergent G^1 smooth bicubic B-spline surface patches with single interior knots over a given arbitrary quad partition of a polygonal model. In that paper, like in all the existing literatures, the G^1 conditions do not adapt for the geometric properties of the quad partition, i.e., the conditions do not reflect the different sizes of adjacent B-spline surface patches. In this paper, based on the geometric properties of the quad partition, we provide a new local scheme of constructing convergent G^1 smooth bicubic B-spline surface patches with single interior knots over a given arbitrary quad partition of a polygonal model. Our numerical results show that, for the portion jointed by two distinctly different size B-spline surface patches, the new method improves both the shape and the continuity qualities of the surface model significantly.

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Polynomial cellular neural networks for studying complex systems

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In this short communication we study a new neural network architecture, i.e. polynomial cellular neural network (PCNN) for studying complex systems such as reaction-diffusion systems. Reaction-diffusion type of equations are widely used to describe phenomena in different fields, as biology-Fisher model, FitzHugh-Nagumo nerve conduction model, Vector-disease model, chemistry - Brusselator model, physics - Sine-Gordon model, etc [3]. In a recently proposed VLSI development [3] a first CNN based hardware implementation with polynomial weight functions has been presented.

Cellular neural networks (CNN) [1], [2], introduced by L.Chua and L.Yang in 1988, have the basic application in image processing. They are novel class of information processing systems constructed as an analog dynamic processor array which reflects just this property: the processing elements interact directly within a finite local neighborhood. Reaction-diffusion CNN can virtually represent any feature of reaction-diffusion systems. Moreover, the existence of locally active cells in such networks is a necessary condtion for emergent complex behaviour.

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Vector Shock Soliton Solution of Wave Equation in Three Space Dimension By Bilinear Transformation Method

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The Hirota bilinear method is applied to find exact solitary wave solutions of the vector wave equation in three dimension for n-component vector order parameter, with reaction part in the form of the third order polynomial determined by three distinct constant vectors. The bilinear representation is derived by extracting one of the vector roots, which allows us reduce cubic nonlinearity to the quadratic one. Shock soliton solution, implementing transition between other two roots, as a fixed points of the potential from the continuum set of values, is constructed in a simple way. The velocity of solitary wave is found in terms of these three roots. Extensions of this model, by including the the gradient term to the equation is also studied and solitary waves are derived. The numerical solutions illustrating generation of solitary waves in special cases are given.

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The measure of the power of a voter and his capacity to influence the voting outcome

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A simple game which is a mathematical formulation of some voting situations consists of a set N of voters together with a set of winning coalitions. A winning coalition S enjoys absolute power. This means that if the members of S reach an agreement then their decision is implemented irrespective of what the other voters do. In such games, several power concepts have been defined in the literature. To name only a few, we have the Shapley-Shubik [4] and the Banzhaf-Coleman's [1] indices, and the Taylor's influence relation [5].

Diffo et al [2, 3] unfold the problem of the true nature of the power modelled by these concepts. Since true power must traduce the voter's capacity of really influencing the voting outcome so as to enforce his personal interests, the major problem is the follow : if a theory of power assigns to some voter j more power than to someone i, does it means that in a voting situation, one can observe that j is more than i capable to influence the voting outcome?

Diffo and Moulen [2] proved that if individual preferences are linear orders then

- the Taylor's influence relation traduces well the capacity to influence the voting outcome,

- the Shapley-Shubik's and the Banzhaf-Coleman's indices traduce the capacity to influence the voting outcome if and only if the game is trade-robust.

We assume that abstentions are allowed. We show that the results above are often false, we obtain a necessary and sufficient condition in which the power concepts above traduce the capacity to influence the voting outcome.

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Patient-dependent parameter: a mathematical model

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Analytical methods for predicting and exploring treatment strategies of diseases dynamics have proven to have useful applications in public health policy and planning. We derive the patient-dependent parameter from an age-physiology dependent population model. This parameter is an important concept in biomedical sciences.

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The Bernoulli-Navier model for beams as limit of the Kirchhoff-Love model for plates

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It is well known that the Kirchhoff-Love theory for linearly elastic plates with sufficiently small thickness consists of a mathematical model that involves three partial differential equations posed over the middle surface: a fourth order differential equation for the transverse displacement, and a system of two second order differential equations for the in-plane displacements. This model was justified from mathematical point of view by Ciarlet[1] by an asymptotic analysis (cf. Lions[2]) from three dimensional elasticity model, taking the thickness as a small parameter. In the same manner, using the area of the cross-section as the small parameter, Trabucho and Viaño[2] justified the Bernoulli-Navier model for elastic beams. This model is written as three one-dimensional differential equations posed over the middle line: two forth order differential equations for the transversal displacements (bending) and one second order differential equation for axial displacement (extension).

In this paper we study the asymptotic behavior of the solution of the Kirchhoff-Love theory for a rectangular plate $\Omega^{\varepsilon t} = S^t \times (-\varepsilon, \varepsilon)$, with middle surface $S^t = (0, L) \times (-t, t)$, when its width t tends to zero. After a change of variable to a domain that does not depend on t we show that the limit, in a well defined sense, is the classical Bernoulli-Navier extension-flexural theory for the rectangular beam $\Omega^{\varepsilon t} = (0, L) \times \omega^{\varepsilon t}$, of length L and transversal cross section $\omega^{\varepsilon t} = (-t, t) \times (-\varepsilon, \varepsilon)$.

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On arbitrary boundary conditions in Maxwell equations for microwave circuit characterization

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The numerical solution of the time-harmonic Maxwell equations by the Finite Element Method gives rise to sparse linear systems with a rather large number of unknowns, especially in three-dimensional domains. The goal is to achieve an efficient computation of such systems. In this context, arbitrary boundary conditions allow us to analyze different microwave circuits by means of the same sparse linear system solution. Defining a suitable basis for the tangential electric and magnetic vectorial fields on the boundary, a generalized admittance matrix is obtained. In addition to the basis defined in [1], a basis of eigenmodes for accessible waveguides, we add a piecewise curl-conforming vectorial functions basis for the portion of boundary we want to leave arbitrary. This way, a Padé approximation scheme for the wide-band response of the circuit in the frequency domain via the Lanczos process [2] is still applicable. Furthermore, the arbitrary boundary conditions can be used to obtain a domain decomposition method [3], as the proposed basis makes it possible to divide a domain into an arbitrary number of subdomains. Practical applications are presented to illustrate the capability of this strategy.

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