Characterizations of the beta distribution on symmetric matrices

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The real beta distribution with parameters $p > 0$ and $q > 0$ may be presented as the distribution of the random variable $U/U+V$, where $U$ and $V$ are two independent random variables with gamma distributions $\gamma_{p,\sigma}$ and $\gamma_{q,\sigma}$ respectively, it is given by

$$ \beta_{p,q}(dx) = \frac{x^{p-1}(1-x)^{q-1}}{B(p,q)} 1_{(0,1)}(x) dx $$

where $B(p,q)$ is the beta Euler function.

Similarly, as the multivariate version of the gamma distribution is the Wishart distribution on the cone of positive definite symmetric $(r,r)$-matrices or on any symmetric cone $\Omega$, the multivariate beta random variable is defined as the “quotient” of some Wishart random variables, using a division algorithm (see Hassairi, Lajmi and Zine (2005)), it is given by

$$ \beta_{p,q}(dz) = (B_\Omega(p,q))^{-1}(\Delta(z))^{p-\frac{r+1}{2}}(\Delta(e-z))^{q-\frac{r+1}{2}} 1_{\Omega \cap (e-\Omega)}(z) dz,$$

where $e$ is identity matrix, $\Delta(z)$ is the determinant of $z$ and $B_\Omega(p,q)$ is the beta function of the symmetric cone $\Omega$.

Recently Seshadri and Wesolowski (2003) have given two characterization results concerning the real beta distribution based on some constancy of regression conditions. For instance, let $X$ and $Y$ be two independent, non-degenerate random variables in $(0,1)$ and let $V = 1 - XY$ and $U = (1 - Y)/V$. The first result says that if

$$ \mathbb{E}(U \mid V) = c \text{ and } \mathbb{E}(U^2 \mid V) = d, \tag{1} $$

where $c$ and $d$ are real constants, then $(X,Y) \sim \beta_{p,q} \otimes \beta_{p+q,s}$ and consequently $(U,V) \sim \beta_{r,q} \otimes \beta_{r+q,p}$, where $q = \frac{(1-c)[d-c]}{c^2-d} > 0$, $s = \frac{c(d-c)}{c^2-d} > 0$ and $p > 0$.

The second result says that if

$$ \mathbb{E}(U \mid V) = c \text{ and } \mathbb{E}(U^{-1} \mid V) = b, \tag{2} $$

where $c$ and $b$ are real constants, then $(X,Y) \sim \beta_{p,q} \otimes \beta_{p+q,s}$ and consequently $(U,V) \sim \beta_{r,q} \otimes \beta_{r+q,p}$ where $q = \frac{(b-1)(1-c)}{(bc-1)} > 0$, $s = \frac{c(b-1)}{bc-1} > 0$ and $p > 0$. 

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$p > 0$. In the present work, we first establish some properties of the matrix Beta distribution, in particular, we give the distribution of some generalized powers and we show the independence of some related statistics. We then give multivariate versions of the regression formulas (1) and (2) and we show that each of the two extended versions characterizes the beta-Wishart distribution on symmetric matrices.

References


Combining Edgeworth’s and Berry–Esseen’s approaches to the central limit theorem

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We obtain near optimal Berry–Esseen bounds in the classical case. This is achieved by distinguishing the lattice and the nonlattice cases, as one–term Edgeworth expansions do. The main tool is an easy inequality involving the usual second modulus of continuity, in substitution of Esseen’s smoothing inequality.

To be more precise, let \((X_k)_{k \geq 1}\) be a sequence of iid random variables such that \(EX_1 = 0\), \(EX_1^2 = \sigma^2 > 0\) and \(E|X_1|^3 = \beta < \infty\). Denote by \(F_n\) the distribution function of \((X_1 + \cdots + X_n)/(\sigma \sqrt{n})\) and by \(\Phi\) the standard normal distribution function. Depending on whether or not \(F_n\) is lattice, we prove that

\[
\frac{1}{2} D(F_n) \leq \|F_n - \Phi\| \leq \frac{1}{2} D(F_n) + \frac{\beta}{6\sigma^3 \sqrt{2\pi}} (1 + \delta)n^{-1/2} + O(n^{-1}), \quad (1)
\]

or

\[
\|F_n - \Phi\| \leq \frac{\beta}{6\sigma^3 \sqrt{2\pi}} (1 + \delta)n^{-1/2} + O(n^{-1}), \quad (2)
\]

respectively, where \(\| \cdot \|\) stands for the usual supremum norm, and \(D(F_n)\) for the maximum jump of \(F_n\). In (1)–(2), \(\delta\) is a positive parameter as close as we wish to zero and the “big o” terms are explicitly bounded, the upper bounds depending on some parameters arbitrarily chosen in certain ranges. No numerical computations are needed.

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References


Stochastic differential equations with fractional noise

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Let \( B_t = (B^1_t, \ldots, B^n_t) \) where the processes \( B^j, j = 1, \ldots, m \) are independent fractional Brownian motion (fBm) with Hurst Parameter \( H > \frac{1}{2} \). Consider the following \( d \)-dimensional stochastic integrals:

\[
X_t = X_0 + \int_0^t g(X_s) dB_s + \int_0^t f(X_s) ds
\]

where the integral with respect to \( B \) is a pathwise Riemann-Stieltjes integral of type Zahle[3]. This kind of stochastic integral equation has been studied by several authors, see for example Lin[2], Nualart and Rascanu[1]. In [1] existence and uniqueness of the solution is proved where

\( (g1) \) \( g \) satisfies Lipschitz nonlinearity,

\( (g2) \) partial derivative of \( g \) satisfies local Holder continuity properties,

\( (g3) \) there exist \( \gamma \) in \([0,1]\) and \( K_0 > 0 \) such that for all \( x \) in \( \mathbb{R}^d \)

\[
\|g(x)\| \leq K_0(1 + \|x\|^\gamma),
\]

\( (f1) \) \( f \) satisfies Lipschitz continuity,

\( (f2) \) \( f \) has sublinear growth.

In this paper we cover a more general situation, for the above integral equation assumig that \( f \) is semimonotone i.e.

\[
< f(x) - f(y), x - y > \leq \|x - y\|^2.
\]

An existence and uniqueness result is proved, which relies heavily on Picard iteration method of Zangeneh[4].
References


Geometrical probabilities of Buffon type in bounded lattices

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The story began in 1733 when Comte de Buffon was asking for the probability that a needle which is dropped at random on a set of equidistant parallel lines in the plane intersects one of them. For the case that the lines are $a$ units apart and the needle is of length $l < a$ Buffon gave the solution $p = \frac{2l}{\pi a}$ in 1777. In the following years and centuries based on results of Crofton, Cartan and Poincaré and other mathematicians the theory of geometric probability was born and developed, for an excellent overview see [4]. In the last decade a lot of general examinations where considered like [2] and [3] varying the lattice and the objects of test elements.

In this work now we will go back to Buffon and consider in the first part a bounded lattice and unbounded test elements i.e. Buffon’s needle problem will be inverted. As well we will show some results for bounded lattices and bounded test elements in form of line segments which generalizes the theorem of Diaconis for long needles, [1]. For that we will look for needles $H$ which have to intersect the convex hull $K$ of the finite lattice $C$, i.e. $H \cap K \neq \emptyset$, and furthermore under the restriction that they have to fall completely into $K$, $H \subset K$. For both scenarios we will present the geometric probabilities $p_k$ of exactly $k = 0, 1, ..., m, m+1$ intersections of the needles $H$ with the lattice $C$ of equidistant parallel line segments of length $d$ and $a$ units apart consisting of $m$ elementary cells. For the case $H \subset K$ a nice recursive formula for the $p_k$ is derived which gives us an explicit function for $p_k$. In all our results we will find a structure of the form $p_k = f(k-1) + g(k) + h(k+1)$ with functions $f, g, h : \mathbb{N} \rightarrow \mathbb{R}$, some times with a big number of case distinctions. In spite of this the formula for the mean value $N$ of intersections between the needles and the lattice based on the distribution $p_k$ in the case of the scenario $H \cap K \neq \emptyset$ is very comprehensible: $N = \frac{\frac{a}{d} + 1}{\frac{a}{d} + \frac{2}{\pi} \frac{l}{a} + \frac{1}{2}}$. Finally we will look at some computer based experiments finding the geometrical probabilities numerically.

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Design of continuous regression tests by transforming the accumulated residuals process

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Transformed empirical processes have been used since [1] (see [2] and nested references) in the construction of goodness-of-fit tests, consistent for all alternatives, focused on the sequences of contiguous alternatives chosen by the user, and asymptotically distribution free under the null hypothesis of fit as well as under the alternatives of focusing. After transforming, these limiting laws are the same, regardless there are estimated parameters or not.

In this work we consider sequences of linear models with continuous regressors \( x_h, h = 0, 1, \ldots, p - 1 \), that, eventually changing variables to obtain a normalized form, is written as

\[
Y_{n,j} = \sum_{h=0}^{p-1} \hat{\beta}_h x_h(j/(n+1)) + \sigma Z_{n,j}, Z_{n,j} \text{ i.i.d. } (j = 1, \ldots, n), EZ_{n,1} = 0, EZ_{n,1}^2 = 1.
\]

With suitable consistent estimators \( \hat{\beta}_{n,h}(h = 0, \ldots, p - 1) \), \( \hat{\sigma} \) of the parameters, the estimated residuals are \( \hat{e}_{n,j} = \hat{\sigma}_n^{-1}(Y_{n,j} - \sum_{h=0}^{p-1} \hat{\beta}_h x_h(j/(n + 1))) \). We introduce the process of accumulated estimated residuals (AER) \( \hat{r}_n(t) = \sum_{j=1}^{[nt]} \hat{e}_{n,j} \) that under infill asymptotics \( (n \to \infty) \), converges in law to a standard Wiener process on \([0,1]\).

The same kind of transformation applied to the empirical process in the articles referred above is applied in the present work to the AER process, leading to a transformed accumulated residual process that is also asymptotically Gaussian, from which Cramér-von Mises tests statistics of Watson type ([3]) are constructed, thus providing tests which are also consistent under all alternatives, can be focused to the alternatives of interest of the user, and have distribution free asymptotic laws.

As a particular application, a simple test for polynomial regression exploiting properties of the Legendre Polynomials is constructed and its behaviour is described empirically.
References


Free-differentiability conditions on the free-energy function implying large deviations

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In this talk we will present our recent results concerning a problem of large deviation in the real line ([1]). Let \((\mu_a)\) be a net of Radon sub-probability measures on \(R\) and \((\alpha \alpha)\) be a net in \([0, +\infty[\) converging to 0. Let \(C(R)\) denote the set of \([-\infty, +\infty]\)-valued continuous functions on \(R\). For each \(h \in C(R)\), we define \(\Lambda(h) = \log \lim \inf \mu_{\alpha \alpha}(e^{h/\alpha})\) and \(\overline{\Lambda}(h) = \log \lim \sup \mu_{\alpha \alpha}(e^{h/\alpha})\) where \(\mu_{\alpha \alpha}(e^{h/\alpha})\) stands for \(\int_R e^{h(x)/\alpha} \mu_{\alpha}(dx)/\alpha\), and write \(\Lambda(h)\) when both expressions are equal. For each pair of reals \((\lambda, \nu)\), let \(h_{\lambda,\nu}\) be the function defined by \(h_{\lambda,\nu}(x) = \lambda x\) if \(x \leq 0\) and \(h_{\lambda,\nu}(x) = \nu x\) if \(x \geq 0\) (we write simply \(h_\lambda\) in place of \(h_{\lambda,\lambda}\)). For each real \(\lambda\), we put \(L(\lambda) = \Lambda(h_\lambda)\) when \(\Lambda(h_\lambda)\) exists.

A well-known problem of large deviations in \(R\) (usually stated for sequences of probability measures) is the following ([5], pp. 48): assuming that \(L(\lambda)\) exists and is finite for all \(\lambda\) in an open interval \(G\) containing 0, and that the map \(L|_G\) is not differentiable on \(G\), what conditions on \(L|_G\) do imply large deviations, and with which rate function?

In relation with this problem, R. S. Ellis posed the following question ([4]): assuming that \(\Lambda(h_{\lambda,\nu})\) exists and is finite for all \((\lambda, \nu)\) in \(R^2\), what conditions on the functional \(\Lambda|_{\{(h_{\lambda,\nu}: (\lambda, \nu) \in R^2\}}\) do imply large deviations with rate function \(J(x) = \sup_{(\lambda, \nu) \in R^2} \{h_{\lambda,\nu}(x) - \Lambda(h_{\lambda,\nu})\}\) for all \(x \in R\)?

In this paper, we solve the above problem by giving conditions on \(L|_G\) involving only its left and right derivatives; the rate function is obtained as an abstract Legendre-Fenchel transform \(\Lambda|_{S^*}\), where \(S\) can be any set in \(C(R)\) containing \(\{h_\lambda : \lambda \in G\}\). When \(S = \{h_\lambda : \lambda \in G\}\), we get a strengthening of the G"artner-Ellis theorem by removing the usual differentiability assumption. The answer to the Ellis question is obtained with \(S = \{h_{\lambda,\nu} : (\lambda, \nu) \in R^2\}\).

The techniques used are refinements of those developed in previous author’s works ([2], [3]), where variational forms for \(\Delta(h)\) and \(\overline{\Lambda}(h)\) were obtained.
References


Particle approximations for a class of stochastic partial differential equations

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Let \((\Omega, \mathcal{F}, P)\) be a probability space on which we have defined an \(m\)-dimensional standard Brownian motion \(W\) which drives the following semilinear stochastic partial differential equation

\[
d\vartheta_t(\varphi) = \vartheta_t(A\varphi)dt + \sum_{k=1}^{m} (\vartheta_t(\gamma_k\varphi) - \vartheta_t(\gamma_k)\vartheta_t(\varphi)) + \vartheta_t(B^k\varphi))(dW^k_t - \vartheta_t(\gamma_k)dt).
\]

(1)

Here \(\vartheta\) is a measure valued process, \(A\) is a second order differential operator and \(B^k, k = 1,\ldots, m\) are first order differential operators. Equation (1) is known as the Fujisaki-Kallianpur-Kunita or the Kushner-Stratonovitch equation. It plays a central role in nonlinear filtering: The solution of (1) gives the conditional distribution of a stochastic process \(X\) (the signal) given an associated observation process \(W\) (the process \(W\) becomes a Brownian motion after a suitable change of measure).

The aim of the paper is to show how one can construct a particle approximation to the solution of (1). The main idea is to merge the weighted approximation approach, as presented in Kurtz and Xiong [5] for a general class of nonlinear stochastic partial differential equation to which (1) belongs, with the branching corrections approach introduced by Crisan and Lyons in [2].

The result is a generalization of existing results (see for example [1, 3, 4]) where the differential terms \(\vartheta_t(B^k\varphi), k = 1,\ldots, m\) are missing. The addition of these differential terms in (1) is important. They appear in the case where the signal noise and the observation noise are correlated. This feature is quite common, for example, in financial applications. To our knowledge this is the first particle algorithm that treats the correlated case.

References


Hitting probabilities for systems of non-linear stochastic heat equations

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We consider a system of $d \geq 1$ coupled non-linear stochastic heat equations in spatial dimension 1, driven by $d$-dimensional space-time white noise. The solution of this system is a process indexed by space-time, with values in $\mathbb{R}^d$. The main objective is to determine, for a given subset of $\mathbb{R}^d$, whether or not this set is hit by the space-time process. Using Malliavin calculus, we obtain in [1, 2] upper and lower bounds on the univariate and bivariate joint densities of the solution. This leads to upper and lower bounds on hitting probabilities for the space-time process, in terms of capacity and Hausdorff measure of the sets. We also obtain related estimates when one of the space-time parameters is fixed. This makes it possible to determine the critical dimension above which points are polar, as well as the Hausdorff dimensions of the range of the process and of its level sets. Similar results were obtained in [3] for systems of stochastic wave equations in spatial dimension 1.

References


Sequential analysis under composite hypotheses

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Let \( \Theta_i \in R^n, i = 0, 1, \Theta_0 \cap \Theta_1 = \emptyset, \Theta_0 \cup \Theta_1 = \Theta \) are compact sets, \( f(x, \theta_i), \theta_i \in \Theta_i \) are density functions (d.f.) w.r.t. some \( \sigma \)-finite measure \( \mu \), for \( \mu \)-a.s. \( x \) \( f(x, \cdot) \) is continuous and positive. The problem of sequential testing of hypotheses \( \{H_i : \theta_i \in \Theta_i\}, i = 0, 1 \) based on i.i.d.r.v. \( \{x_k\} \) with d.f. \( f \) is considered. Put \( \eta(\theta) = \frac{\ln f(x, \theta_i)}{f(x, \theta_i)}, i \neq j, i, j = 0, 1 \). Suppose that \( \forall u = (\theta_i, \theta_j, \theta_j^*), i \neq j, i, j = 0, 1 \exists \{u\}(u) > 0 \) s.t. \( \mathbb{E}_{\theta_i} \exp (t(u)\eta(\theta)) < \infty, \mathbb{E}(\eta(\theta)|H_i, \theta_j^* \in \Theta_i) \) and modulus of continuity of \( D(\eta(\theta)|H_i, \theta_j^* \in \Theta_i) \) are positive and small enough. Let \( \Lambda_i > 0, i = 0, 1, \) and define the stopping times (st.t) \( T_i = \inf \{n : \min_{\theta \in \Theta} \sum_{k=1}^{n} \ln \frac{f(x_k, \theta_i)}{f(x_k, \theta_j)} > \) and \( T = T_0 \cup T_1, \) The decision rule is: to accept \( H_i \) at instant \( T \) if \( T = T_i \).

**Theorem 1.** If \( \Lambda_i, i, j = 0, 1, i \neq j \) is large enough then \( \forall \) st.t. \( \tau \) with given constraints on maximal (over \( \theta \)) error probabilities the inequalities hold: \( \max_{\theta_i \in \Theta_i} \mathbb{E}(\tau|H_i, \theta_i \in \Theta_i) \leq \max_{\theta_i \in \Theta_i} \mathbb{E}(\tau|H_1, \theta_0 \in \Theta_i) \).

This result is a generalization for a composite hypotheses of classical Wald’s result [1]. Denote by \( \mathbb{P}_{m, \theta}(\mathbb{E}_{m, \theta}) \) the measure (mathematical expectation) corresponding to a sequence \( \{x_k\}_{k=1}^{\infty} \) with the change-point (c-p) [2] at the instant \( m \): the d.f. of \( x_n \) is equal to \( f(x, \theta_0), n < m \) and \( f(x, \theta_1), n \geq m \). The problem consists in sequential detection of a c-p \( m \). All known methods of c-p detection contain some ”large parameter” \( N \) (f. e., the threshold). For any \( \theta_i^* \in \Theta_i \) define \( \theta_i^*(\cdot) = (\theta_i^*(\cdot), \theta_j^*(\cdot)) \) : \( \max_{\theta_i \in \Theta_i} \left( \ln \frac{f(x, \theta_i^*)}{f(x, \theta_j^*)} \right) = I_{0i}^{-1}(\theta_i^*(\cdot), \theta_j^*) \). Put \( \theta^* = (\theta_i^*(\cdot), \theta_j^*) \) and define the false alarm probability \( \alpha_{\theta^*}^{a}(\theta^*) = \sup_{n} \mathbb{P}_{\infty, \theta^*}(d_n^a(n) = 1) \) for any method \( a \) with ”large parameter” \( N \) (here \( d_n^a(n) = 1(d_n^a(n) = 0) \) corresponds to the decision about the presence (absence) of a change at the instant \( n \).

**Theorem 2.** Let \( \tau_n^n = \min\{n : \mathbb{E}_{\theta^*}(\tau_n^n) = 1\} \). If \( \forall \theta^* \in \Theta, \forall m \exists \lim_{N \rightarrow \infty} \mathbb{E}_{m, \theta^*}(\tau_n^n) < \infty, \lim_{N \rightarrow \infty} \frac{\ln \alpha_{\theta^*}^{a}(\theta^*)}{N} > 0 \) then \( \lim_{N \rightarrow \infty} \frac{\mathbb{E}_{m, \theta^*}(\tau_n^n) + \ln \alpha_{\theta^*}^{a}(\theta^*)}{N} \geq I_{0i}^{-1}(\theta^*) \).

A method of sequential c-p detection is called asymptotically optimal (a.o) if the last inequality turns into a strict equality (the left-hand side can be proposed as a natural performance index for c-p problems). Put \( L(n, \theta) = \max_{1 \leq k \leq \infty} \sum_{k=1}^{n} \ln \frac{f(x_k, \theta_i)}{f(x_k, \theta_j)} \).
and \[ T_N^* = \inf \{ n : \min_{\theta_0 \in \Theta_0} \max_{\theta_1 \in \Theta_1} \mathcal{L}(n, \theta) > N \} \].

**Theorem 3.** The detection method corresponding to the st.t. \( T_N^* \) is a.o.

This result is a generalization of classical CUSUM [3] procedure for the case of composite hypothesis not only after but also before the c-p.

**References**


Wiener chaos solutions of linear forward-backward stochastic differential equations

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Forward-backward stochastic differential equations (FBSDEs) have attracted the attention of the stochastic analysis community on account of their interesting mathematical theory and their widespread applicability in areas such as control theory or mathematical finance.

Starting from a recent construction of solutions for linear stochastic differential equations, by Mikulevicius and Rozovskii (see e.g. [1]), which uses the Wiener chaos expansion, we propose a new scheme for the solution of linear forward-backward stochastic differential equations using the decomposition of the solution in a chaos expansion. Through duality arguments, we propose a variational formulation of FBSDEs which when coupled with the chaos expansion provides a constructive way of solving the system.

References

Probability on independent probability spaces

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In [1], an earlier paper by the author (publication pending), an algebra structure called a reliability algebra is introduced. A reliability algebra is a set $M$ together with two binary operations $\bullet$ and $\oplus$ and a unary operation $\overline{\cdot}$ satisfying:

$R_1 : (M, \bullet)$ is a commutative monoid,

$R_2 : \overline{\overline{a}} = a$ for $a \in M$,

$R_3 : \overline{a \oplus b} = \overline{a} \bullet \overline{b}$ for all $a, b \in M$ (De Morgan’s law).

The use of reliability algebra for finding the reliability of an organizational system and the conditions necessary for a reliability algebra to be a Boolean Algebra [1] is discussed in the paper. The functioning components placed in series or parallel can be expressed as a member of the reliability system $M = [0, 1]$ with $a \oplus b = a + b - a \bullet b$ and unary $\overline{\cdot}$ defined by $\overline{a} = 1 - a$.

This paper generalized paper [1] in finding expressions for the probability of more general events expressed in terms of a finite number of independent events $\{E_i\}_{i=1}^n$ of probabilities $\{p_i\}_{i=1}^n$.

We define an independent probability space in terms of basic events $\{E_i\}_{i=1}^n$ and values $\{p_i\}_{i=1}^n$ in $[0, 1]$. We also define a reliability algebra. One can view the probability measure as a function from the reliability algebra $B[E_1, \ldots, E_n]$ of Boolean polynomials into the reliability algebra $M[p_1, \ldots, p_n]$ of polynomials in $\{p_i\}_{i=1}^n$. Conditions under which the probability measure $P$ is a homomorphism from $B[E_1, \ldots, E_n]$ into $M[p_1, \ldots, p_n]$ is discussed and that $P$ maps symmetric functions into symmetric function is proven. We develop formulas for the image of every symmetric function under $P$.

References


A tool for detecting customers’ leaving based on time series models

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Nowadays, customer fidelity is a very important issue within the marketing strategy of any company (i.e. banking or communication business). Therefore, research on the behavior of customers is a critical task to succeed.

In this work, our aim is to provide marketing researchers with a tool that allows them to follow customers’ path along the time, in a single way. We propose a tool based on time series models which is intended to represent the evolution of the fidelity level of each customer. Our proposal defines a new combined variable that will be taken as the index to be considered and predicted by the time series model.

The predictions given by the model for each customer at several moments of time in the future are considered in order to determine if that customer is at risk of leaving. In this case, suitable retention commercial actions over that customer are suggested to be performed by the company.

References


On a characterization of the generalized gamma distribution

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The generalized gamma distribution is a fairly flexible family of distribution which includes as special cases the exponential, Weibull, gamma, and log-normal distributions. They are widely used and play an important role in the reliability field and the survival analysis, therefore a successful estimation of its parameters will be very important.

Ten years ago, we at first do research on gamma distribution, using a non-linear transformations published in Reference 1, and concentrated on its characterization. The results: The independence of sample coefficient of variation, sample Gini’s mean difference and sample range \( R_n \) and sample mean \( \bar{X}_n \) respectively are equivalent to gamma distribution are obtained. More results are presented in the References 2, 3, 4, and 5.

Since same result holds for generalized gamma distribution can be concluded with same procedure using for gamma distribution, therefore more results are obtained such as the p.d.f., mean, variance, skewness and kurtosis of the coefficient of sample range \( R_n/\bar{X}_n \) for small sample under gamma and its application to quality control. Further research will be concentrated on with the similar topics mentioned above under Weibull distribution in the near future.

References


Robustness in statistical forecasting of time series

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Statistical forecasting is intensively used to solve many significant applied problems. Optimality of the majority of the known statistical forecasting procedures is proved w.r.t. the mean square risk of prediction under the assumptions of the underlying hypothetical model. In practice, however, the observed time series satisfy the hypothetical models with distortions of some level $\varepsilon$ [1]. Unfortunately, the forecasting procedures, which are optimal under hypothetical models, often lose their optimality and become unstable under “a little distorted hypothetical models”. That is why the following main problems considered in this paper are very topical: A) robustness evaluation of the traditionally used statistical forecasting procedures under distortions; B) evaluation of $\delta$-admissible distortion levels; C) construction of new robust forecasting procedures by the minimax risk criterion.

By the method of asymptotic expansions of the risk functional w.r.t. $\varepsilon$ [2, 3] we solve these problems for the following cases:

- trend models of time series under “outliers” and functional distortions of trends;
- multivariate linear regression models under “interval” distortions, “relative” distortions and distortions in $l_p$-metrics;
- autoregressive time series under non-homogeneities in the innovation process and under bilinearities;
- vector autoregressive time series under “misspecifications”;
- vector autoregressive time series under missing values.

Theoretical results are illustrated by the results of computer experiments on simulated and real time series.

References


Large deviations of dependent discrete random variables

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Let $\xi_1, \xi_2, \ldots$ are independent Poisson variables of means $\lambda_1, \lambda_2, \ldots$. Following [1], we consider for $n > 1$ random variables $\bar{\mu}_n = (\frac{\mu_1}{n}, \frac{\mu_2}{n}, \ldots, \frac{\mu_n}{n})$ such that

$$P\{\mu_\nu = k_\nu, \nu = 1, \ldots, n\} = P\{\xi_\nu = k_\nu, \nu = 1, \ldots, n\, | \sum_{\nu = 1}^{n} \nu \xi_\nu = n\}.$$

Consider the function $\rho(\bar{x}, \bar{y}) = \sup_\nu |\ln \frac{x_\nu}{y_\nu}|$, where $0 \ln 0, \ln \frac{0}{0}$ are taken to be 0 and $|\ln \frac{1}{0}, |\ln 0|$ are taken to be $\infty$. It is easy to prove that function $\rho(\bar{x}, \bar{y})$ is a metric which may reach infinite value on

$$\Omega = \{\bar{x} = (x_1, \ldots), x_\nu \geq 0, \nu = 1, 2, \ldots, \sum_{\nu = 1}^{\infty} \nu x_\nu = 1\}.$$

Let function $\phi(\bar{x})$ be continuous on space $\Omega$ with metric $\rho(\bar{x}, \bar{y})$ and for any $\bar{x} \in \Omega$

$$\lim_{n \to \infty} \phi(x_1, \ldots, x_n, 0, 0, \ldots) = \phi(\bar{x}).$$

Then for any sequence $a_n$ that tends to $a$ as $n \to \infty$

$$\lim_{n \to \infty} \frac{1}{n} \ln P\{\phi(\bar{\mu}_n) > a_n\} = J(\bar{\lambda}, \{\bar{x} \in \Omega, \phi(\bar{x}) > a\}) - J(\bar{\lambda}, \Omega),$$

where $\bar{\lambda} = (\lambda_1, \lambda_2, \ldots)$, $J(\bar{x}, \bar{y})$ is Kullback-Leibler divergence [2],

$$J(\bar{x}, \bar{y}) = \sum_{\nu = 1}^{\infty} x_\nu \ln \frac{x_\nu}{y_\nu}, \quad J(\bar{x}, A) = \inf_{\bar{y} \in A} J(\bar{x}, \bar{y}).$$

References


The configurational measure on mutually avoiding SLE paths

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2000 Mathematics Subject Classification. 60G50, 60J45, 60J65

We discuss a new approach to the study of multiple chordal SLEs in a simply
connected domain by considering configurational measures on paths instead
of infinitesimal descriptions. We construct these measures so that they are
conformally covariant, and satisfy boundary perturbation and Markov prop-
erties, as well as a cascade relation. As an application of our construction,
we derive the scaling limit of Fomin’s identity [1] in the case of two paths
directly.

Theorem (Kozdron-Lawler): If \( \gamma : [0, \infty) \to \mathbb{H} \) is an SLE\(_2\) in the upper
half plane \( \mathbb{H} \) from 0 to \( \infty \), and \( \beta : [0, 1] \to \mathbb{H} \) is a Brownian excursion from
\( x \) to \( y \) in \( \mathbb{H} \) where \( 0 < x < y < \infty \), then

\[
P\{ \gamma[0, \infty) \cap \beta[0, 1] = \emptyset \} = 1 - \frac{H(f(0), f(y)) H(f(x), f(\infty))}{H(f(0), f(\infty)) H(f(x), f(y))}
\]

where \( f : \mathbb{H} \to \mathbb{D} \) is a conformal transformation of the upper half plane
\( \mathbb{H} \) onto the unit disk \( \mathbb{D} \), and \( H(z, w) \) is the excursion Poisson kernel in \( \mathbb{D} \)
given by

\[
H(z, w) := H_{\partial \mathbb{D}}(z, w) := \frac{1}{\pi} \frac{1}{|w - z|^2} = \frac{1}{2\pi} \frac{1}{1 - \cos(\arg w - \arg z)}.
\]

A complete discussion of the configurational measure including the proof
of the above theorem may be found in [4] which extends the work done in [3].
Further details about the Brownian excursion measure may be found in [2],
and for more information about SLE\(_2\) and its relationship to loop-erased
walk see [5]. This talk is based on joint work with Gregory F. Lawler of
Cornell University.

References


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Lévy white noise measures on infinite dimensional spaces

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It is shown that a Lévy white noise measure Λ[1] always exist as a Borel measure on the dual \( K' \) of the space \( K \) of \( C^\infty \) functions on \( \mathbb{R} \) with compact support. Then a characterization theorem that ensures that the measurable support of \( \Lambda \) is contained in \( S' \) is proved. In the course of the proofs, a representation of the Lévy process as a function on \( K' \) is obtained and stochastic Lévy integrals are studied. The results is a key to extend [2] to a more general class of Lévy white noise functionals, such as functionals associated with stable processes.

References


A mechanical model of Markov processes

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We consider the motion of several massive particles (molecules) in an ideal gas of identical point particles (atoms) in a $d$-dimensional Euclidean space, with certain interactions. It is well-believed that the motion of the molecules converges to a Markov process when the mass $m$ of atoms converges to 0, heuristically because the central limit theorem for “independent identically distributed” atoms. However, the atoms are actually not independent, since they could be the ones interacted with some molecule(s) already.

This problem was first considered by Holley, in the case that the particles are moving in a one dimensional space. The corresponding question for general dimensional case has been considered by, e.g., [1], [2], but for the system of collision with only one molecule.

In this study, we consider the problem of multi-molecules without the independent assumption (which, as explained, actually does not hold). We prove the existence of the solution of the corresponding equation, and study the limit when $m$ converges to 0.

References


An analysis of the last round matching problem

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The matching problem or the “Hats Problem” goes back to at least 1713 when it was proposed by the French mathematician Pierre de Montmort in his book [1] on games of gambling and chance. Detailed progress made on this problem can be found in [2] and [3]. In this paper, we consider the multi-round matching problem: Suppose that each of \( n \) men at a party throws his hat into the center of the room. The hats are first mixed up, and then each man randomly selects a hat. Suppose that those choosing their own hats depart, while the others put their selected hats in the center of the room, mixed the hats up, and then reselect. Also, suppose that this process continues until each individual has his own hat. Assume we start with \( n \) men. We study the probability distribution \( L_n \) of the number of players in the last round of the matching problem and obtain the existence of the limiting distribution by using convolution method. We give the recursive relations for the expectation \( E(L_n) \) and estimate the distribution \( P(L_n = m) \). We show that the limiting distribution of \( L_n \) exists and the expectation \( E(L_n) \) converges to a constant \( l \approx 2.26264703816 \).

References


Optimum designs for the description of sorption data using isotherm equations

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2000 Mathematics Subject Classification. 62K05

Gas-solid adsorption can be described phenomenologically in terms of an adsorption function \( n = f(P,T) \) where \( n \) is the amount adsorbed, \( P \) the pressure and \( T \) the temperature at which the process occurs. As a matter of experimental convenience, the adsorption isotherm is determined by \( n = f_T(P) \) and in a detailed study this is done for several temperatures. Adsorption isotherms are widely used to determine the surface area and pore size distribution of a variety of different solid materials, to characterize retention of chemicals in soils, to model the water transport occurring in foods during processing, and many other industrial processes. Therefore a correct characterization is crucial for the best quality of many goods.

Simple equations, such as Langmuir, BET [1] or GAB [2] are commonly used to describe adsorption data. Certain parameters in these equations are considered independent from \( P \) and their estimation has a great importance for the characterization of the mentioned phenomena.

In order to improve the accuracy of the parameters estimations, optimum designs have been discussed [3]. In this work \( T \)–optimum designs maximizing the differences between competing equations have been calculated to discriminate. Besides, for each equation, \( D \)–optimum designs minimizing the determinant of the covariance matrix of the estimates and \( c \)–optimum designs to individually estimate a parameter have been found. All the optimum designs have been checked using the Equivalence theorem.

Traditional used common experimental designs have been compared to the optimum designs obtained, and as result, a valuable tool has been provided for researches to choose the most appropriate design compared to an optimum with the help of the efficiency concept.

References

Incomplete samples from stationary sequences and extreme values

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We consider a strictly stationary random sequence \( (X_n) \) with the marginal distribution function \( F(x) = P\{X_1 \leq x\} \). Suppose that some of random variables \( X_1, X_2, X_3, \ldots \) can be observed, and let \( \varepsilon_k \) be the indicator of the event that random variable \( X_k \) is observed and \( S_n = \varepsilon_1 + \cdots + \varepsilon_n \). Let us denote \( \tilde{M}_n = \max\{X_j : 1 \leq j \leq n, \varepsilon_j = 1\} \), and \( M_n = \max\{X_1, \ldots, X_n\} \). Suppose that d.f. \( F \) belongs to the maximum domain of attraction of some of extreme value distributions. The limiting distribution of the random vector \( (\tilde{M}_n, M_n) \) and ”asymptotic independency” of \( \tilde{M}_n \) and \( M_n \) are obtained under a condition imposed on asymptotic behavior of the sum \( S_n \) and a condition of weak dependency of random variables from the sequence \( (X_n) \).

References


Stochastic comparisons of Jones-Larsen’s model

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Recently, in [1], a general family of distributions for the empirical modelling of ordered multivariate data is proposed. This model extends the joint distribution of order statistics from an independent and identically distributed univariate sample. This model was considered first by [5] and [2] from a theoretical point of view. Let us consider $n+1$ real positive parameters $\mathbf{a} = (a_1, \ldots, a_{n+1})$, and consider the joint density

$$f_{\mathbf{a}}(x_1, \ldots, x_n) = \frac{\Gamma(a_1 + \cdots + a_{n+1})}{\prod_{j=1}^{n} \Gamma(a_j)} \left\{ \prod_{j=1}^{n} f(x_j) \right\} \times \prod_{j=2}^{n} \{F(x_j) - F(x_{j-1})\} F(x_1)(1 - F(x_n)),$$

on $x_1 < x_2 < \cdots < x_n$, where $F$ is a continuous distribution function and $f$ the corresponding density function.

The purpose of this work is to provide conditions for the stochastic comparison of two of these models. In particular, given two joint densities as above, based on distribution functions $F$ and $G$, we provide conditions on $F$ and $G$, to compare in stochastic, likelihood ratio and hazard rate orders (see [3] and [4]), the corresponding random vectors or the marginal distributions.

References

Limit theorems for sequences of blockwise negatively associated random variables

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2000 Mathematics Subject Classification. 60F15

In this paper, we defined blockwise negatively associated random variables and prove Marcinkiewicz strong law and classical strong law of large numbers for a sequence of blockwise negatively associated random variables.

References


A problem of stochastic approximation in genetics

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We refer to an application to genetics discussed especially by W. Feller, D. Ludwig, M. Mangel, Z. Schuss, A.T. Bharucha-Reid; also some problems of convergence are discussed by G.V. Orman. Now we emphasize some aspects concerning the approximation of Markov chains by a solution of a stochastic differential equation to determine the probability of extinction of a genotype. Thus, the Markovian nature of the problem will be pointed out again, and we think that this is a very important aspect.

Let us consider a population of $N$ individuals consisting of $X_N(n) = i$ individuals of type $A$ in the $n$th generation. Then the next generation consists of $N$ individuals randomly selected from a practically infinite offspring of the previous generation. Obviously, the selection process is binomial and the probability of extinction of a genotype, or the time until extinction become very hard to calculate.

For this reason the Markov chain $\{X_N(n)\}$ can be approximated by a diffusion process, or more exactly, by a solution of a stochastic differential equation. We come to a problem of convergence and it is shown that the convergence is sufficiently rapid as to satisfy some imposed conditions provided that a specific stochastic differential equation has a unique solution. Thus, the existence and uniqueness of the solution are shown.

A natural conclusion is found again: if $x(t)$ is the solution of a specific stochastic differential equation with absorbing boundaries at $x=0$ and $x=1$, then the probability of extinction is the probability of exit of $x(t)$ from the interval $(0,1)$.

Remark. Obviously, various situation may exist when the survival of a particular genotype can be very dynamic. In general, the interaction of a population can have a great complexity, which lead to the enhancement of the interdisciplinary coordination in these studies.

References


Bayesian sample size determination for a \(2 \times 2\) contingency table with dependent proportions

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2000 Mathematics Subject Classification. K62c10

Several measures used in the analysis of a \(2 \times 2\) contingency table are based on the two conditional probabilities, \(\pi_{1|1}\) and \(\pi_{1|2}\), with \(\pi_{1|1}\) and \(\pi_{1|2}\) dependent, or their ratio \(\theta = \pi_{1+}/\pi_{1+}\). They can also use the relative risk associated with rows \(\rho\), defined by \(\rho = \frac{\pi_{1|1}}{\pi_{1|2}} = \frac{\pi_{11}}{\pi_{21}} / \frac{\pi_{12}}{\pi_{22}}\), or the odds ratio \(\psi = \frac{\pi_{1|1}}{\pi_{2|1}} / \frac{\pi_{1|2}}{\pi_{2|2}} = \frac{\pi_{11}}{\pi_{21}} / \frac{\pi_{22}}{\pi_{12}}\). In a Bayesian context, we consider the vector \(\pi = (\pi_{11}, \pi_{12}, \pi_{21}, \pi_{22})\), with \(\sum_{i=1}^{2} \sum_{j=1}^{2} \pi_{ij} = 1\). Let the prior distribution of \(\pi\) be a Dirichlet distribution, denoted \(\pi \sim \text{Dir}(\alpha_1, \alpha_2, \alpha_3, \alpha_4)\) with \(\alpha_i > 0\), \(0 \leq \pi_i \leq 1\), \(i = 1, \ldots, 4\), and density \(f(\pi_1, \pi_2, \pi_3) = \prod_{i=1}^{3} \pi_i^{\alpha_i-1} / B(\alpha_1, \alpha_2, \alpha_3, \alpha_4)\), defined in the simplex \(\sum_{i=1}^{3} \pi_i \leq 1\).

Sample size determination: We now proceed in two steps:

A) " Mapping out " the positive axis:
   a) First, for each measure, we compute the \((1 - \varepsilon)100\%\) hpd interval for \(\zeta\), using the expression of its posterior density \(f_\zeta(\alpha_1, \alpha_2, \alpha_3, \alpha_4; \{\mathbf{x}\})\).
   b) Secondly, for each of the above measures, we compute the expectation of this length:
   \[K_\zeta(\n) = E_X [\ell_\zeta(\alpha, \beta; n, \{\mathbf{x}\})],\]
   using the marginal Multinomial-Dirichlet distribution of \((X_1, X_2, X_3, \ldots)\).
   c) Thirdly, we let \(n\) vary from 1 to 30. We now have a curve \(C^\zeta_\n\), with vertical coordinate \(K_\zeta(\n)\), function of \(n\), with 0 \(\leq n \leq N\).

B) Determination of the size \(n\): Since each \(K_\zeta(\n)\) is an decreasing function of \(n\), considered here as a continuous variable, if \(\eta_1\) is the desired value of the average precision, we only need to numerically solve:
   \[K_\zeta(\n) \leq \eta_1,\]
   and, in most cases, we have an interval open to the right as solution. Our numerical application concerns well-known Salk’s vaccine results.

References


Generalized Beta distributions, random continued fractions and
Thomae invariance

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2000 Mathematics Subject Classification. 60E05

The aim of this paper is to generalize the following well known result about
Gamma and Beta distributions. If the independent random variables
$X$ and $G_1$ are distributed as Beta $\beta_{a,a'}$ and Gamma $\gamma_{a+a',1}$, respectively, then
$XG_1 \sim \gamma_{a,1}$. Furthermore, if the random variable $G_2$ is distributed as $\gamma_{a',1}$
and it is independent of $(X,G_1)$, then
\begin{align}
G_2 + XG_1 & \sim \gamma_{a+a',1}, \\
\frac{G_2}{G_2 + XG_1} & \sim \beta_{a',a}.
\end{align}

and these two random variables are independent. Defining the random variable
\[ W' \equiv \frac{G_1}{G_2} \sim \beta_{a+a',a}' \]

the beta law of the second kind $\beta_{b,a}^{(2)}$ being the law of the ratio of two
Gamma variables with common scale parameter and shape parameters $b$ and $a$, respectively, we can reexpress (2) in the following way. If $W'$ has the
law (3) and it is independent of $X \sim \beta_{a,a'}$, then
\[ \frac{1}{1 + W'X} \sim \beta_{a',a} \]

Moreover, if $W \sim \beta_{a+a',a}'$ is independent of $(X,W')$, then
\[ \frac{1}{1 + \frac{W}{1+W'X}} \sim X. \]

This relation (5) characterizes the law $\beta_{a,a'}$ [1]. A similar characterization
holds for the generalized inverse Gaussian distribution [2]. In this paper
we generalize this result to the general case $W \sim \beta_{b,a}^{(2)}$, $W' \sim \beta_{b,a'}^{(2)}$, with
$b > 0$ not necessarily equal to $a + a'$. We prove that there is again a unique
law (parametrized by $(a,a',b)$) for $X$, supported by the positive real line,
which ensures the equality in law (5). This result allows a probabilistic
interpretation for a result due to Thomae (contained e.g. in [3]) concerning
the symmetry of certain hypergeometric functions.
References


A comparative analysis of different allotment methods for preferential vote

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We show different proportional methods for preferential vote and we give an analysis of them. These methods can be used for unipersonal or pluripersonal elections. We study their behaviour and some properties like Condorcet, proportionality, manipulation, and so on. Then, we propose applications to political elections and we obtain the winners.

References

On maxima and sum of random number of random variables

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2000 Mathematics Subject Classification. 60F

In this article, limit distributions of sum and maxima of random number of i.i.d. random variables are studied. We consider several cases when the random summand is (i) Geometric, (ii) Negative Binomial, (iii) Poisson, (iv) Discrete Uniform. Under the Poisson case, the random summand does not play any role in the limit distribution. Interesting limit laws arise and we study these limit laws and their properties. An iterative result which is also an invariance principle is obtained in the sequel. Domains of attraction of the limit laws are also characterized.

Keywords and phrases: Random Maxima, Limit Distributions, Geometric, Negative Binomial, Poisson, Discrete Uniform, random summands, Iterative property, Invariance Principle, Domains of Attraction, Extreme Value Theory, Stable Laws.

References


On rough paths and stochastic PDEs

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2000 Mathematics Subject Classification.

This talk is based on the preprint [2], which is a continuation of [1]. We aim at a pathwise definition of stochastic PDEs, based on rough path type techniques inspired by [3]. We consider then an equation on a given separable Hilbert space $B$, of the form

$$dy_t = Ay_t + f(y_t) \, dx_t, \quad y_0 = \psi$$

for $t \in [0,T]$, where $A$ is the generator of an analytical semi-group on $B$, $f : B \to B$ is a regular coefficient, $\psi$ a smooth enough initial condition, and $x$ is a general irregular control, which is supposed to be $\gamma$-Hölder continuous in time, with $\gamma > 1/3$. We show that, up to some algebraic and analytic manipulations, one can get some existence and uniqueness results for equation (1), provided some operator-valued increments of the type

$$n_{ts} = \int_s^t S(t-u)dx_u S(u-s)$$

can be defined, with a given space-time regularity. These results are applied to the basic example of the stochastic heat equation in dimension 1 driven by an infinite dimensional fractional Brownian motion.

References

Some results of generalized mixtures of Weibull distributions

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Weibull distributions and their mixtures play a great role in the reliability theory to model lifetime and failure time data, since they can incorporate wide varieties of failure rate functions.

Some extensions of these distributions arise under the formation of series, parallel and other structures of systems, see e.g. [4], which are mixtures allowing negative weights (generalized mixtures). Generalized mixtures extend the used models to describe the probabilistic behavior, providing more flexibility in their estimations.

Moreover, the generalized mixtures of Weibull distributions with common shape parameter can be considered as extensions of the generalized mixtures of exponentials. In the literature, generalized mixtures of exponential distributions have been characterized and related results have been obtained (among others, see [1], [2], [3] and [5]).

Nevertheless, the generalized mixtures of Weibull distributions have not been studied before. Therefore, in this work, we prove some results for that an arbitrary finite mixture of Weibull distributions with common shape parameter to be a valid probability model.

References


Growth of Lévy trees

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2000 Mathematics Subject Classification. 60J80


More precisely, we define a growing family of discrete Galton-Watson trees with independent identically exponentially distributed branch lengths that is consistent under Bernoulli percolation on leaves; we define the Lévy tree as the limit of this growing family with respect to the Gromov-Hausdorff topology on metric spaces. This elementary approach notably includes supercritical trees and does not make use of the height process introduced by Le Gall and Le Jan to code the genealogy of (sub)critical continuous-state branching processes and used in subsequent publications, notably [2] to study Lévy trees. The special case of Brownian trees was studied in [5], where the growing family of binary Galton-Watson trees was shown to have a property of independent growth increments expressed in form of a composition rule. In the general case of [3] we get a Markovian growth structure.

Using Aldous’s [1] $\ell_1$-representatives of continuum random trees, we construct the mass measure of Lévy trees and we give a decomposition along the ancestral subtree of a Poisson sampling directed by the mass measure.

References


On a successive approximation of the solution for a stochastic differential equation

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In this paper, in a first theorem we give a successive approximation solution of the $d$-dimensional stochastic differential equation:

$$dX(t) = \sigma(t, X(t))dB(t) + \alpha(t, X(t))dt, \quad t \geq 0, \quad X(0) = \xi \in \mathbb{R}^d,$$

where, $d \in \mathbb{N} \setminus \{0\}$, $\sigma : \mathbb{R}_+ \times \mathbb{R}^d \to \mathbb{R}^d \otimes \mathbb{R}^d$ and $\alpha : \mathbb{R}_+ \times \mathbb{R}^d \to \mathbb{R}^d$ are bounded measurable functions satisfying: $\exists \lambda > 0$, such that

$$\forall \delta \in \mathbb{R}^d, \forall t \in [0, +\infty[, \forall y \in \mathbb{R}^d, \delta \star \sigma(t, y) \delta \geq \lambda |\delta|^2.$$

We shall define the approximate solutions $X^n$ for $n \in \mathbb{N} \setminus \{0\}$ by:

$$X^n(t) = \xi \quad \text{on} \quad -1 \leq t \leq 0 \quad \text{and}$$

$$X^n(t) = \xi + \int_0^t \sigma(u, X^n(u - \frac{1}{n}))dB(u) + \int_0^t \alpha(u, X^n(u - \frac{1}{n}))du \quad \text{on} \quad t \geq 0.$$

It shall be shown that

$$\lim_{n \to +\infty} \mathbb{E}\left( |X^n(t) - X(t)|^2 \right) = 0 \quad \text{for any} \quad t \geq 0$$

whenever the pathwise uniqueness of the solution holds.

Compared with the standard Itô’s successive approximation technique, the advantage of this method is: given any $n \in \mathbb{N} \setminus \{0\}$, one can calculate $X^n$ by step-wise iterated Itô integrals over the intervals $[0, \frac{1}{n}], [\frac{1}{n}, \frac{2}{n}], [...], \text{etc.}$ We see there is no need to calculate $X^i(t), i < n$ at all.

In a second theorem, we give a successive approximation solution of a 1–dimensional stochastic differential equation involving the local time in 0 of the unknown process. Our method consists in using a Zvonkin-transformation in order to come back to a stochastic differential equation satisfying the conditions of the first theorem.

We consider that this work pave the way for the resolution of the successive approximations convergence problem of an important class of stochastic differential equations involving the local time of the unknown process.
References


White-noise functionals as infinite dimensional random Colombeau distributions

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It is well-known that the white-noise is not a genuine process, but can be represented as random linear distributions of the form $\Omega \rightarrow \mathcal{D}'$ or as members of the white-noise space $(S', \mathcal{B}, \mu)$, where $S'$ is the space of tempered distributions, $\mathcal{B}$ a related $\sigma$-field and $\mu$ the Bochner measure. However these representations are rather smoothed versions of the white-noise of the form $\xi : S \times S' \rightarrow \mathbb{R}^d$. In order to define $\mathcal{B}(t)$ and its non-linear functions like $[\mathcal{B}(t)]^2$, $e^{\mathcal{B}(t)}$ one has to resort to Hida, Kubo-Takenaka, Kondratiev or some other distributions. These spaces are usually the duals of the test function spaces constructed on $(S', \mathcal{B}, \mu)$. However in all cases the representation of the non-linear functions and functionals of the white-noise is a painstaking process, e.g. in the case of Hida distributions one has to utilize the second quantization operator and renormalization in each separate case.

In this communique, after a critical evaluation of the afore-mentioned distributional methods, an attempt is made of the construction of an infinite dimensional Colombeau distribution $\mathcal{G}(S', \mathcal{B}, \mu)$. In this space, like in $\mathcal{G}(\mathbb{R}^n)$, the algebra operations would be possible; so that the products of the white-noise functions and functionals can be defined in a natural way. This is the extension of the work on random Colombeau distributions as introduced and developed in [1] and [2]. It turns out that the counterparts of Sobolev derivatives, Cameron-Martin formula can be defined, moderate germs $\mathcal{A}$ and null germs $\mathcal{N}$ can be constructed on $S'$, so that an infinite dimensional Colombeau algebra $\mathcal{G}/\mathcal{N}$ can be formulated.

References

