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Short Communications Abstracts

Section 08

Analysis

Jensen–Steffensen's and related inequalities for superquadratic functions

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Jensen–Steffensen's inequality states [2] that if $\varphi\colon I \to \mathbb{R}$ is convex, then

$$\varphi(\bar{\zeta}) \leq \frac{1}{P_n} \sum_{i=1}^{n} \rho_i \varphi(\zeta_i)$$

holds, where I is an interval in $\mathbb{R}$, $\boldsymbol{\zeta} = (\zeta_{1,\ldots,}\zeta_n)$ is any monotonic n-tuple in I^n, $\bar{\zeta}$ is defined as $\bar{\zeta} = \frac{1}{P_n}\sum_{i=1}^{n} \rho_i \zeta_i$, and $\boldsymbol{\rho} = (\rho_1, \ldots, \rho_n)$ is a real n-tuple that satisfies

$$\begin{aligned} &0 \leq P_j \leq P_n\ , \quad j = 1, \ldots, n, \quad P_n > 0, \\ &P_j = \sum_{i=1}^{j} \rho_i, \quad \overline{P}_j = P_n - P_{j-1}, \quad j = 2, \ldots, n. \end{aligned}$$

Slater's Pečarić inequality states [3] that under the same conditions leading to Jensen–Steffensen's inequality if $\sum_{i=1}^{n} \rho_i \varphi'(\zeta_i) \neq 0$ and if

$$M = \frac{\sum_{i=1}^{n} \rho_i \zeta_i \varphi'(\zeta_i)}{\sum_{i=1}^{n} \rho_i \varphi'(\zeta_i)} \in I, \quad \text{then} \quad \sum_{i=1}^{n} \rho_i \varphi(\zeta_i) \leq P_n \varphi(M).$$

In this presentation we refine the above theorems. These refinements are achieved by superquadratic functions.

Definition. [1] A function $\varphi\colon [0,\infty) \to \mathbb{R}$ is *superquadratic* provided that for all $x \geq 0$ there exists a constant $C(x) \in \mathbb{R}$ such that

$$\varphi(y) - \varphi(x) - \varphi(|y-x|) \geq C(xt)(y-x)$$

for all $y \geq 0$.

Theorem. *Under the condition of the theorems above, if φ is nonnegative superquadratic, then*

$$\begin{aligned} P_n\varphi(\bar{\zeta}) &+ (n-1)P_n\varphi\Big(\frac{\sum_{i=1}^{n} \rho_i(|\zeta_i - \bar{\zeta}|)}{(n-1)P_n}\Big) \\ &\leq \sum_{i=1}^{n} \rho_i \varphi(\zeta_i) \leq P_n\varphi(M) - (n-1)P_n\varphi\Big(\frac{\sum_{i=1}^{n} \rho_i(|\zeta_i - M|)}{(n-1)P_n}\Big). \end{aligned}$$

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Partial differential equations related to generating functions

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The Faber polynomials $(F_k)_{k\geq 1}$ are given by the identity

$$1 + b_1 w + b_2 w^2 + \cdots + b_k w^k + \cdots = \exp\big(- \sum_{k=1}^{+\infty} \frac{F_k(b_1, b_2, \ldots, b_k)}{k}\, w^k \big)$$

The polynomials $(K_n^p)_{n\geq 1}$, $p \in Z$ are given by

$$(1 + b_1 w + b_2 w^2 + \cdots + b_k w^k + \cdots)^p = 1 + \sum_{n\geq 1} K_n^p(b_1, b_2, \ldots, b_n) w^n$$

They are homogeneous polynomials of degree n in the variables $(b_1, b_2, \ldots)$ where b_k has weight k. Let $X_0 = -\sum_{j\geq 1} b_j \frac{\partial}{\partial b_j} = -b_1 \frac{\partial}{\partial b_1} - b_2 \frac{\partial}{\partial b_2} - \cdots - b_k \frac{\partial}{\partial b_k} - \cdots$. Then the identity

$$K_n^{-1} = \sum_{0\leq j\leq n} K_j^{-2} K_{n-j}^1$$

which correponds to the multiplication $h(z)^n = h(z)^{n-1} h(z)$ is the same as the partial differential equation on the manifold of coefficients

$$\frac{\partial^2 F_n}{\partial b_r \partial b_s} = \frac{\partial}{\partial b_{r+s}}(X_0 F_n) \qquad \forall r,\ s \geq 1, \quad n \geq 1$$

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Geometric properties of intersection bodies

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Intersection bodies are convex bodies whose radial function is a positive definite distribution. They were introduced in 1988 by Lutwak [3] in connection to the Busseman-Petty problem (see [2], chapter 5, for details). In general, no much is known about the geometry of intersection bodies, even of those that are polytopes.

In 1998, Koldobsky [1] introduced a necessary condition for a convex body to be an intersection body in terms of the second derivative of its norm. This result allowed him to prove that the unit ball of the q-sum of two spaces X and Y is not an intersection body.

In our work we use the techniques of [1] to prove that, in dimension 7 or more, an intersection body cannot be a direct sum of two convex bodies. We also prove that certain bodies of revolution that have a face are not intersection bodies in sufficiently high dimensions.

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On the growth of L^p Lebesgue constants in $\mathbf{R}^d$

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The one dimensional Lebesgue constant of order N, i.e., the L^1 norm over the one dimensional Torus $[0,1]$ of $\sum_{k=0}^{N-1} e(kx)$, where $e(x) = e^{2\pi ix}$, is of the order of $\ln N$. This result can be generalized in several ways, one of which is discussed here. First, the L^1 norm can be replaced by the L^p norm, where $1 < p < \infty$. The order then becomes $N^{1-p^{-1}}$. Next, the dimension can be increased from 1 to d. But this second generalization requires also a generalization of $\{0, 1, \ldots, N-1\}$ to a subset of $\mathbf{Z}^d$. If that subset is simply taken to be $\{0, 1, \ldots, N-1\}^d$, then the order becomes $N^{(1-p^{-1})d}$. This motivates the conjecture that $N^{(1-p^{-1})d}$ should also be the order when that subset is taken to be a d-dimensional polytope containing 0 dilated by N and then intersected with $\mathbf{Z}^d$. This is already known to be true when $d = 2$. [1] When $d > 2$, the conjecture may follow as a straightforward generalization of Belinsky's proof of the corresponding result for $L^1(\mathbf{R}^d)$. [2] We will discuss this conjecture.

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Excesses and deficits of frames in shift-invariant subspaces

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For an invertible $n \times n$ matrix B and Φ a finite or countable subset of $L^2(R^n)$, consider the collection

$$X = \{\phi(. - Bk) : \phi \in \Phi, k \in Z^n\},$$

generating the closed subspace $\mathcal{M}$ of $L^2(R^n)$. Let $T_{\mathcal{F}(X)}(\xi)$ denote the frame operator associated with the frame $\{\mathcal{F}\phi(\xi)\}_{\phi\in\Phi}$ defined for a.e. $\xi \in [0,1)^n$, where $\mathcal{F}$ is the isometric isomorphism between $L^2(R^n)$ and $L^2(T^n, \ell^2(Z^n))$. Using a very nice property of the range function, the Gramian and dual Gramian operators (G and $\tilde{G}$ resp.) and $\mathcal{F}$, we will show that if $\mathcal{M}$ is a Shift-Invariant subspace generated by X, one need at most m functions, where $m = || \dim(Ker(\tilde{G}(.)))||_\infty$, to generate the orthogonal complement of $\mathcal{M}$ in $L^2(R^n)$. Furthermore, if $k \geq m$ or $k = \infty$, one can always find k functions such that the associated Shift-Invariant system form a Parseval tight frame for $\mathcal{M}^\perp$. Finally we will show that the existence of a collection of m sequences in the orthogonal complement of the range of analysis operator associated with the frame X that satisfies any of four interesting conditions is equivalent to $\dim(Ker(G(\xi)))$, the dimension of the kernel of Gramian operator, being less than or equal to m for almost all $\xi \in [0,1)^n$.

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Boundedness of pseudodifferential operators associated with Laguerre hypergroup

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Let $n \in \mathbb{N}$, $\alpha \geq 0$, $\mathbb{K} = [0, \infty[\times \mathbb{R}$ and $\widehat{\mathbb{K}} = \mathbb{R} \times \mathbb{N}$. We denote by $\varphi_{(\lambda,m)}(x,t)$ the unique solution of the following system of partial differential operators:

$$\begin{cases} D_1 & = & \frac{\partial}{\partial t}, \\ D_2 & = & \frac{\partial^2}{\partial x^2} + \frac{2\alpha+1}{x}\frac{\partial}{\partial x} + x^2\frac{\partial^2}{\partial t^2} \ ; \qquad (x,t) \in]0,\infty[\times\mathbb{R}. \end{cases}$$

One knows that $\varphi_{\lambda,m}(x,t) = e^{i\lambda t}\mathcal{L}_m^\alpha(|\lambda|x^2)$, where $\mathcal{L}_m^\alpha$ is the Laguerre functions. We recall here that for $(\lambda, m) \in \mathbb{R}\times\mathbb{N}$ and for a suitable function, the Fourier-Laguerre transform $\mathcal{F}(f)(\lambda,m)$ is given by

$$\mathcal{F}(f)(\lambda,m) = \int_{\mathbb{K}} \varphi_{-\lambda,m}(x,t)f(x,t)d\mu_\alpha(x,t), \quad \text{where} \quad d\mu_\alpha(x,t) = \frac{x^{2\alpha+1}dxdt}{\pi\Gamma(\alpha+1)}.$$

And for reasonable function Ψ defined on $\widehat{\mathbb{K}}$, the inverse of the above Fourier Laguerre transform is given by

$$\mathcal{G}_\alpha(\Psi)(x,t) = \int_{\widehat{\mathbb{K}}} \varphi_{(-\lambda,m)}(x,t)\Psi(\lambda,m)d\gamma_\alpha(\lambda,m),$$

where $d\gamma_\alpha(\lambda,m) = L_m^\alpha(0)\delta_m \bigotimes |\lambda|^{\alpha+1}d\lambda$ is the Plancherel measure on $\widehat{\mathbb{K}}$. For all $\gamma \in \mathbb{R}$ we define the Sobolev-Laguerre type space $\mathcal{H}_p^\gamma$ as the set of all tempered distributions f such that $\mathcal{F}_\alpha f \in L^p_{loc}(\mathbb{K}, dm_\alpha)$ and $\|f\|_{\mathcal{H}_p^\gamma} < \infty$ where

$$\|f\|_{\mathcal{H}_p^\gamma} = \left(\sum_{m=0}^{+\infty} L_m^\alpha(0)\int_{\mathbb{R}} \Big(1 + |\lambda|\Big(m + \frac{\alpha+1}{2}\Big)\Big)^{p\gamma} \Big|\mathcal{F}_\alpha f(\lambda,m)\Big|^p |\lambda|^{\alpha+1}d\lambda\right)^{1/p}.$$

The aim of this paper is to study the continuity of generalized pseudodifferential operator $B_{\alpha,\sigma}$ [3] and the commutator $[B_{\alpha,\sigma}\ ,\ I_\varphi\ ]$ on the Sobolev-Laguerre type-spaces $\mathcal{H}_p^\gamma$ [1], [2] where σ belongs to a class of generalized symbols defined on $\mathbb{K}\times\widehat{\mathbb{K}}$ and $I_\Phi = \mathcal{G}_\alpha\left(\Phi\mathcal{F}_\alpha\left(\cdot\right)\right)$. Φ is being a suitable function.

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Kolmogorov-type inequalities for the derivatives of multivariate functions

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In this talk we shall give a review of known results on the exact Kolmogorov-type inequalities for intermediate derivatives of multivariate functions. Such inequalities are important in many extremal problems of Analysis [1, 2]. In addition, we shall present a series of new inequalities of this type as well as discuss their applications to Approximation Theory. To give an example of the results obtained let us mention an inequality which estimates the L_∞-norm of the mixed derivative of order $\alpha = (\alpha_1, \ldots, \alpha_d)$, $\alpha_j > 0,\ j = 1, \ldots, d$, $\sum\limits_{j=1}^{d} \alpha_j < 1$, in Marchaud [3] sense of the function defined on $\mathbf{R}^d$ in terms of the L_∞-norm of the function itself and the L_∞-norms of the first partial derivatives of the function:

$$\|D^\alpha f\|_\infty \le \frac{2^{d-1}}{\prod\limits_{j=1}^{d} \Gamma(1-\alpha_j)} \cdot \frac{2^{1-\sum\limits_{j=1}^{d}\alpha_j}}{1-\sum\limits_{j=1}^{d}\alpha_j} \cdot \|f\|_\infty^{1-\sum\limits_{j=1}^{d}\alpha_j} \cdot \prod_{j=1}^{d} \left\| \frac{\partial f}{\partial t_j} \right\|_\infty^{\alpha_j}.$$

This inequality is obtained jointly with Sergey Pichugov.

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On a proof of Milovanović et al. conjecture on integrals of fast oscillating functions

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The study of orthogonal polynomials with respect to oscillatory complex weight function was considered in [1], [2]. While Working on a project on orthogonal systems and numerical integration and from some facts on fast oscillating functions, Milovanović, Cvetković and Stanić conjectured that for any ζ positive with the condition that$\sin 2\zeta < 0$, we have

$$I\left(\zeta, k\right) = (-1)^{N(\zeta)} \int_{-1}^{1} t \sin\left(\zeta t\right) \prod_{j\ =1}^{k} \left(t^2 - \left[b\left(\zeta, j\right)\right]^2\right) dt > 0$$

where $N\left(\zeta\right) = \lfloor \frac{\zeta}{\pi} \rfloor$ and $b\left(\zeta, j\right) = \left(N\left(\zeta\right) - j + 1\right) \frac{\pi}{\zeta}$, for $\ j = 1, 2, ..., N\left(\zeta\right).$ In this talk we give a proof of this conjecture and moreover we provide a formula for the integral in question in terms of the n-th derivatives of the polynomial product in the integrand. Furthermore, using Viete's Theorem this integral can be expressed in terms of some shifting of the zeros of this polynomial.

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Fibonacci numbers and orthogonal polynomials

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Let $F_0 = 0, F_1 = 0, \dots$ with $F_{n+1} = F_n + F_{n-1}, n \geq 1$ be the sequence of Fibonacci numbers. We prove that $(1/F_{n+2})$ is the moment sequence of a discrete probability and we identify the corresponding orthogonal polynomials as little q-Jacobi polynomials, cf. [1]

$$p_n(x\phi; q, 1; q) = {}_2\varphi_1 \left(\begin{matrix} q^{-n}, q^{n+2} \\ q^2 \end{matrix} ; q, -\frac{x}{\phi} \right),$$

where $\phi = (1 + \sqrt{5})/2$ is the golden ratio and $q = -1/\phi^2$. The corresponding Hankel matrix $(1/F_{i+j+2})$ was called the Filbert matrix in [2], and it was established via computer algebra that its inverse matrix has integer coefficients expressed in terms of the Fibonomial coefficients

$$\binom{n}{k}_{\mathbb{F}} = \prod_{i=1}^{k} \frac{F_{n-i+1}}{F_i}, \quad 0 \leq k \leq n.$$

We prove that

$$F_{n+1} p_n(x\phi; q, 1; q) = \sum_{k=0}^{n} (-1)^{kn - \binom{k}{2}} \binom{n}{k}_{\mathbb{F}} \binom{n+k+1}{n}_{\mathbb{F}} x^k,$$

and that the corresponding kernel polynomials for the orthonormal polynomials have integer coefficients. This explains the result of Richardson. A similar but more elementary result holds for the Hilbert matrix $(1/(i+j+1))$, which is the Hankel matrix for Lebesgue measure on $[0, 1]$, and the corresponding orthogonal polynomials are Legendre polynomials.

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Discretization and transference for bisublinear maximal operators

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We discretize M. Lacey's strong type boundedness results for the bisublinear Hardy-Littlewood maximal operators and for the bisublinear maximal Hilbert transform (see [3]), obtaining the counterpart of each of these results for the sequence spaces ℓ^p. We then give some transference applications of the discretized versions to maximal estimates and almost everywhere convergence in Lebesgue spaces of abstract measures deriving, in particular, an expanded range of exponents for the a.e. convergence in Bourgain's double recurrence theorem (see [2]). The techniques used here extend those introduced in [1] for the case of the bilinear Hilbert transform.

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A geometric characterization: complex ellipsoids and the Bochner-Martinelli kernel

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This talk concerns symmetry properties of certain Cauchy-Fantappiè kernels that arise in the study of holomorphic functions in complex Euclidean space. First, Boas' characterization of bounded domains for which the Bochner-Martinelli kernel is self-adjoint is extended to the case of weighted boundary measure. For strictly convex domains, this equivalently characterizes the ones whose Leray-Aĭzenberg kernel is self-adjoint with respect to weighted boundary measure. In each case, the domains are complex linear images of a ball, and the measure is the Fefferman measure. Finally, the Leray-Aĭzenberg kernel for a strictly convex hypersurface in $\mathbb{C}^n$ is shown to be Möbius invariant when defined with respect to Fefferman measure.

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Classification of hyperbolicity and stability preservers

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A linear operator T on $\mathbb{C}[z]$ is called *hyperbolicity-preserving* or an *HPO* for short if $T(P)$ is hyperbolic whenever $P \in \mathbb{C}[z]$ is hyperbolic, i.e., it has all real zeros. One of the main challenges in the theory of univariate complex polynomials is to describe the monoid $\mathcal{A}_{HP}$ of all HPOs. This outstanding open problem goes back to Pólya-Schur's well-known characterization of multiplier sequences of the first kind, that is, HPOs which are diagonal in the standard monomial basis of $\mathbb{C}[z]$. Pólya-Schur's 1914 result generated a vast literature on this subject and related topics at the interface between analysis, operator theory and algebra but so far only partial results under rather restrictive conditions have been obtained. In this talk we report on the progress towards a complete solution of this problem and its multivariate versions made by the authors in an ongoing series of papers.

The concepts of hyperbolicity and (Hurwitz) stability have natural multivariate extensions: a polynomial $f \in \mathbb{C}[z_1, \ldots, z_n]$ is *stable* if $f(z_1, \ldots, z_n) \neq 0$ for all $(z_1, \ldots, z_n) \in \mathbb{C}^n$ with $\Im(z_j) > 0$, $1 \leq j \leq n$. A stable polynomial with real coefficients is called *real stable*. Hence a univariate real stable polynomial is hyperbolic in the above sense. We generalize the notion of multiplier sequences to multivariate polynomials and give a complete characterization of higher-dimensional multiplier sequences. We then classify all operators in the Weyl algebra $\mathcal{A}_n$ of differential operators that preserve stability and show that real stability preservers in n variables are generated by real stable polynomials in $2n$ variables via the symbol map. One of the key ingredients in the proofs is a natural duality theorem for the Fischer-Fock space in n dimensions $\mathcal{F}_n$ that we establish in the process: an operator in $\mathcal{A}_n$ preserves stability if and only its Fischer-Fock adjoint does. This is a powerful generalization of the classical Hermite-Poulain-Jensen theorem in the univariate case as well as a natural multivariate extension of the latter. For $n = 1$ we thus obtain complete algebraic and geometric descriptions of the monoid $\mathcal{A}_{HP} \cap \mathcal{A}_1$ and we further describe all monotone HPOs on $\mathbb{C}[z]$, which solves the aforementioned problem in essentially all nondegenerate cases. Moreover, we prove an analog of the Lax conjecture for real stable polynomials and as a consequence we deduce a (third) determinantal characterization of operators in $\mathcal{A}_{HP} \cap \mathcal{A}_1$. These results have particularly interesting applications to the spectral theory of exactly solvable operators and the Heine-Stieltjes problem for differential equations of Lamé type.

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The algebra of differential operators associated to a family of matrix valued orthogonal polynomials

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The subject of matrix valued orthogonal polynomials was introduced around 1950 by M. G. Krein. If one is considering possible applications of these polynomials it is natural to concentrate on those cases where some extra property holds. In particular, the problem of studying those that satisfy second order differential equations was raised by A. Duran in the nineties (see [2]) and studied by several people, including A. Duran, A. Grünbaum, I. Pacharoni and J. Tirao, who introduced several new examples of this situation (see [3], [4], [5]). As more examples become available, one can expect to see a large body of matrix valued polynomials satisfying second order differential equations and ready to be used in diverse fields of mathematics and its applications.

The present paper (see [1]) represents a change of emphasis from the previous work in this subject. Consider a *fixed* family of orthogonal matrix polynomials P_n that are common eigenfunctions of *some* differential operator L with matrix coefficients and a *matrix valued* eigenvalue Λ_n, $P_nL = \Lambda_n P_n$, $n \geq 0$. We study *the algebra of all such differential operators going along with the family* P_n. The problem is explored through a detailed look at some explicit examples.

Whereas in the scalar case (usually connected with the names of Hermite, Laguerre and Jacobi) this algebra is commutative and has one generator, the examples given in this paper point to a very rich picture: each example discussed here behaves in a different fashion. In each example we display the generators of the (generally non-comutative) algebra, and we give explicit polynomial relations satisfied by these generators.

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The multiplier for the interval $[-1,1]$ related to the Dunkl transform on the real line

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Given $d\mu_\alpha(x) = (2^{\alpha+1}\Gamma(\alpha+1))^{-1}|x|^{2\alpha+1}\,dx$, the Dunkl transform on the real line is given by

$$\mathcal{F}_\alpha f(y) = \int_{\mathbb{R}} f(x) E_\alpha(-iyx)\,d\mu_\alpha(x), \quad y \in \mathbb{R},$$

where E_α is a function that can be expressed in terms of the Bessel function of the first kind (see [1, 2, 3]). In the particular case $\alpha = -1/2$, we have $E_{-1/2}(\lambda x) = e^{\lambda x}$, and $\mathcal{F}_{-1/2}$ is the classical Fourier transform.

Via the Schwartz class, the $[-1,1]$-multiplier $\mathcal{M}_\alpha$ is defined, with the usual notation, as

$$\mathcal{F}_\alpha(\mathcal{M}_\alpha f)(x) = \chi_{[-1,1]}(x)\mathcal{F}_\alpha f(x).$$

We have studied the boundedness of the operator $\mathcal{M}_\alpha$ in weighted L^p spaces (in the case $\alpha = -1/2$, $\mathcal{M}_{-1/2}$ is the so-called ball multiplier for the Fourier transform, which is bounded in $L^p(\mathbb{R}, dx)$ for $1 < p < \infty$).

Our main result analyzes the sufficient and necessary conditions to obtain the inequality

$$\|\mathcal{M}_\alpha f w_{a,b}\|_{L^p(\mathbb{R},d\mu_\alpha)} \le C\|f w_{a,b}\|_{L^p(\mathbb{R},d\mu_\alpha)},$$

where $w_{a,b}(x) = |x|^a(1+|x|)^{b-a}$. As a simple consequence, we have

$$\|\mathcal{M}_\alpha f\|_{L^p(\mathbb{R},d\mu_\alpha)} \le C\|f\|_{L^p(\mathbb{R},d\mu_\alpha)} \iff \tfrac{4(\alpha+1)}{2\alpha+3} < p < \tfrac{4(\alpha+1)}{2\alpha+1}.$$

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New results on conformal mapping of multiply connected domains

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This presentation will survey a number of recent developments, aimed at applications, in the construction of explicit formulae for conformal mappings to multiply connected domains. This will include new formulae for mappings to multiply connected polygonal domains (i.e., a multiply connected Schwarz-Christoffel map [1]) as well as mappings to multiply connected domains with circular arc boundaries (joint work with A. S. Fokas). The mathematical construction employs a Schottky model of the Schottky double of conformally equivalent circular domains and all formulae are expressed in terms of a prime function defined thereon.

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On a question of A. M. Davie on bounded approximation

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The following question was formulated in [1] (p. 127): *let U be a bounded open subset of the complex plane, and let A be a set of bounded analytic functions on U. Which bounded analytic functions on U are limits of bounded sequences of functions in A converging pointwise in U?* This question has been addressed (and solved) only in the particular case where A consists of the polynomials [2]. Let X be a Hausdorff compact and let $C(X)$ be the Banach space of all complex continuous functions f on X with sup norm $||f||_X$. Let B be a closed subspace of $C(X)$ and let $B^\perp$ be the set of all regular complex Borel measures μ on X such that $\int f d\mu = 0$ for every $f \in B$. We suggest the following necessary and sufficient answer to above question even in its more general (abstract) setting.

Theorem 1. *Suppose q is a bounded function on X^o (the interior of X). In order that there exist a sequence $\{u_m\}$, $u_m \in B$, $||u_m||_X \leq M$, and $\lim_{m\to\infty} u_m = q$ pointwise on X^o, it is necessary and sufficient that there exists a sequence $\{q_m\}$, $q_m \in C(X)$, $||q_m||_X \leq M$ such that $\lim_{m\to\infty} q_m = q$ pointwise on X^o and $\lim_{m\to\infty} \int q_m d\mu = 0$ for every $\mu \in B^\perp$.*

Theorem 1 implies in particular the peak-interpolation theorem of E. Bishop (cf. [3], p. 135). This is a new alternative simple proof of Bishop's theorem.

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WSP, finite type and OSC

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We present the relationships between the different separation properties such as the open set condition (OSC), weak separation property (WSP) and finite-type, in the context of graph-directed self-similar iterated function systems. We introduce the notions of a topological weak separation property (tWSP), geometric WSP (gWSP) and Condition M. These definitions apply naturally to the strongly connected components of a general graph. We show that 1. component OSC $\not\Rightarrow$ OSC;
2. component tWSP $\not\Rightarrow$ tWSP;
3. component gWSP $\not\Rightarrow$ gWSP;
4. OSC $\Rightarrow$ tWSP $\Rightarrow$ gWSP but both converses are false;
5. finite-type $\Rightarrow$ gWSP;
6. finite type $\Rightarrow$ Condition M;
7. finite type $\not\Rightarrow$ OSC;
8. finite type $\Rightarrow$ component OSC.

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Privalov's theorem for singular integrals on non-doubling measure metric spaces

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In 1916 Privalov poved that the Conjugate Function is a bounded operator on Lipschitz Spaces of order $s, 0 < s < 1$. In this talk, in the setting of a metric space with a measure μ satisfying the growth condition $\mu(B_r) \leq cr^n$, which allows in particular non-doubling measures, we give a necessary and sufficient condition ("T1" type theorem) for the boundedness of Singular Integral Operators associated to the measure μ, on Liptschitz Spaces of order $s, 0 < s < 1$, defined on the support of μ.

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Existence and uniqueness theorems for fractional differential equations

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Fractional calculus [1, 2] deals with derivatives and integrals of arbitrary order; origin of which stems from the pioneering works of Leibniz, Euler, Abel, Fourier, Riemann, Liouville, and many others [1, 2]. In the present work analysis of fractional differential equations and several existence and uniqueness theorems have been proved [3, 4]. It has been shown that solution of the system of fractional differential equations: $[D^{\alpha_1}y_1, \cdots, D^{\alpha_n}y_n]^t = A(y_1, \cdots, y_n)^t, y_i(0) = c_i$, $i = 1, \cdots n,$, $A = [a_{ij}]$ a real square matrix, turns out to be $\overline{y}(x) = \mathcal{E}_{(\alpha_1,\cdots,\alpha_n),1}(x^{\alpha_1}A_1, \cdots, x^{\alpha_n}A_n)\,\overline{y}(0)$, where $\mathcal{E}_{(\alpha_1,\cdots,\alpha_n),1}$ denotes multivariate Mittag-Leffler function defined for matrix arguments and A_i is the matrix having i-th row as $[a_{i1} \cdots a_{in}]$, and all other entries are zero.

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Operators which commute with the conjugation operators

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Let G be a locally compact group with left invariant Haar measure and let $L^p(G)$, $1 \leq p < \infty$, be the complex Lebesgue spaces associated with it. One of the main results implies that if T is a bounded linear operator from $L^p(G)$ into $L^p(G)$, then T commutes with the conjugation operators, that is, $T({}_y f_y) = {}_y T(f)_y$ for all $f \in L^p(G)$ and $y \in G$ if and only if T commutes with convolution, i.e., $T(\phi \star f) = \phi \star T(f)$ for all $f \in L^p(G)$ and $\phi \in L^1(G)$. We also prove that if T is a weak*-weak* continuous linear operator from $L^\infty(G)$ into $L^\infty(G)$, then T commutes with the conjugation operators if and only if T commutes with convolution.

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Variable exponent Sobolev spaces

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Variable exponent spaces have been intensly studied during the last decade. These spaces form a natural generalization of classical Sobolev spaces to the case when the underlying set or the problem under consideration is not homogeneous. Applications proposed include "intelligent" (electrorheological) fluids [5] and image restoration [2]. In this presentation I will give a brief overview of these applications and of some recent theoretical advances, related to $p(x)$ harmonic functions and the density of smooth functions [1, 3, 4].

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Geometric measure theory on metric structures

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We study geometric measure theory on metric structures. There are two main directions of research:

1) the mappings of Euclidean Space into an *arbitrary* metric space;

2) the mappings of two Carnot–Carathéodory spaces (CC-spaces).

In 1) we study the Lipschitz mappings $\varphi : E \to \mathbb{X}$, $E \subset \mathbb{R}^n$, with $\dim\ker(MD(\varphi, x)) = n - k$ a. e. where $MD(\varphi, x)$ is a metric differential of φ at x. For such mappings, we obtained that a preimage of every point $\varphi^{-1}(z) \setminus \Sigma$, $\mathcal{H}^n(\Sigma) = 0$, is $\mathcal{H}^{n-k}$-rectifiable set. This statement is a base for the next result: *for the validity of the coarea formula for a mapping φ, it is necessary and sufficient for the image $\varphi(E) \subset \mathbb{X}$ to consist of an $\mathcal{H}^k$-rectifiable metric space, and of an image of an $\mathcal{H}^n$-measure zero set.* As a consequence, we obtained that the coarea formula is valid for the mappings with $\mathcal{H}^k$-σ-finite image.

All these results are generalized for mappings defined on $\mathcal{H}^n$-rectifiable metric space $\mathbb{Y}$. Note that the coarea formula for Lipschitz mappings $\varphi : \mathbb{Y} \to \mathbb{R}^k$ was proved by L. Ambrosio and B. Kirchheim in [1].

The problem about necessary and sufficient conditions is essentially new even for $\mathbb{X} = \mathbb{R}^m$, $m, n \geq k$. The short communications can be seen in [2], [3].

In 2) we study geometric properties of CC-spaces and apply them to problems of the geometric measure theory. We introduce the notion of sub-Riemannian coarea factor and *prove the coarea formula for sufficiently smooth contact mappings of two CC-spaces.* This result is also essentially new.

The previous results concerning the coarea formula, were obtained only for different cases of $\mathbb{R}^k$-valued mappings. For the first time, it was proved by P. Pansu for functions defined on a Heisenberg group [4]. Later this result was generalized by J. Heinonen for the function defined on Carnot groups [5], and recently, V. Magnani proved the coarea formula for $\mathbb{R}^k$-valued mappings defined on a Carnot group.

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Univalent Functions Starlike with respect to Infinity in the Positive Direction of the Real Axis

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We characterize univalent functions starlike in the direction of the positive real axis with respect to infinity whose range is contained in a horizontal strip of minimal width $2\alpha\pi$ and contains a horizontal strip of maximal width $2\beta\pi$, where $0 < \beta \leq \alpha \leq \infty$ and $\beta < \alpha$ if $\alpha = \infty$; see [1], [2], [3], and [4].

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On a Sard Type Theorem for C^1-smooth Functions of Two Variables

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Denote by $\operatorname{Int} E$ the interior of a set E, by $\operatorname{Cl} E$ the closure of a set E. One of our main results is as follows.

Theorem 1 (Sard Type Theorem) [2]. *Let $v : \Omega \to \mathbb{R}$ be a C^1-smooth function of a domain $\Omega \subset \mathbb{R}^2$. Suppose $0 \notin \operatorname{Cl}\operatorname{Int}\nabla v(\Omega)$. Then* $\operatorname{meas} f(Z_v) = 0$, *where Z_v is the set of critical points: $Z_v = \{z \in \Omega \mid \nabla v(z) = 0\}$.*

Using Theorem 1 we prove the following result.

Theorem 2 [3]. *Let $v : \Omega \to \mathbb{R}$ be a C^1-smooth function of an open set $\Omega \subset \mathbb{R}^2$. Suppose*

$$\operatorname{Int}\nabla v(\Omega) = \emptyset.$$

Then for any point $z \in \Omega$ there exists a straight line $L \ni z$ such that $\nabla v \equiv \mathrm{const}$ on the connected component of the set $L \cap \Omega$ containing z.

Corollary 3. *Let $b : \mathbb{R}^2 \to \mathbb{R}$ be a C^1-smooth bump. Then the gradient range $\nabla b(\mathbb{R}^2)$ is regularly closed, i.e., $\nabla b(\mathbb{R}^2)$ equals the closure of its interior.*

Recall that a C^1-smooth function $b : \mathbb{R}^n \to \mathbb{R}$ is called *a bump* if its support, defined as the closure of the set $\{z \in \mathbb{R}^n \mid b(z) \neq 0\}$, is bounded and nonempty. In the paper [1] the assertion of Corollary 3 was proved under certain additional conditions on the modulus of continuity of ∇b.

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Strongly l_p-summing m-linear operators

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In this paper, we introduce a new concept consisting in the strongly l_p-summing and r-dominated m-linear operators in the category of operator spaces. We give some characterizations of these such as the Pietsch domination theorem. Some properties and comparison are show.

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Fractional differentiation for Hermite, Laguerre and Jacobi expansions. Applications

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Let us consider the gaussian measure $\gamma_d(x) = \frac{e^{-|x|^2}}{\pi^{d/2}}$ with $x \in \mathbb{R}^d$ and the Ornstein Uhlenbeck differential operator $L = \frac{1}{2}\triangle_x - \langle x, \nabla_x \rangle$. In [1], we define the fractional derivate of order $0 < \alpha < 1$, $D^\gamma_\alpha = (-L)^{\alpha/2}$, induced by the Ornstein Uhlenbeck operator and associated with respect to the gaussian measure and we write, for f a polynomial,

$$D^\gamma_\alpha f = \frac{1}{\Gamma(-\alpha)} \int_0^\infty t^{-\alpha-1}(P_t f - f)dt,$$

where $\{P_t\}_{t\geq 0}$ is the Poisson Hermite semigroup. We obtain a characterization of the gaussian potential $L^p_\alpha(\gamma_d)$ spaces, for $1 < p < \infty$, in terms of this operator. Also, we obtain a version of Calderon's reproduction formula for the gaussian measure in terms of this fractional derivate. A similar characterization of potential spaces using Littlewood Paley theory, was obtained in [2]. When the Laguerre differential operator, $\mathfrak{L}^\lambda = \sum_{i=1}^d x_i \partial^2_{x_i} + (\lambda_i + 1 - x_i)\partial_{x_i}$ with $\lambda \in (-1, \infty)^d$ and the probabilistic gamma measure $\mu^\lambda_d(x) = \Gamma(\lambda + 1)^{-1} \prod_{i=1}^d x^{\lambda_i} e^{-x_i}$. $x \in (0, \infty)^d$ are considered, we introduce fractional derivates in a similar way in [3]. By means relate operators to the fractional derivates, we can define, Triebel Lizorkin spaces and Besov spaces for the Poisson Hermite and the Poisson Laguerre semigroup in [4]. Finally, some results can be extended to the Jacobi setting, considering the Jacobi differential operator $\mathcal{L}^{\alpha,\beta} = \sum_{i=1}^d [(1-x_i^2)\partial^2_{x_i} + (\beta_i - \alpha_i - (\alpha_i + \beta_i + 2)\, x_i)\partial_{x_i}]$ where $\mu_{\alpha,\beta}(dx) = \prod_{i=1}^d (1 - x_i)^{\alpha_i}(1 + x_i)^{\beta_i} dx_i$ is the Jacobi measure, $x \in [-1, 1]^d$ and $\alpha, \beta \in (-1, \infty)^d$.

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Approximation by Nörlund Means of Walsh-Fourier Series in Besov Orlicz Space

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In 1992 Móricz and Siddiqi [3] studied approximation by Nörlund means of Walsh-Fourier series of a function in L_p and, in particular, in $Lip(\alpha, p)$ over unit interval $[0,1]$, where $\alpha > 0$ and $1 \leq p \leq \infty$. This study embraces earlier results in this area by Yano, Jastrebova, and Skvorcov on the rate approximation by Cesáro means, see for example [5]. The problem of characterization of the the Favard (saturation) classes of the Cesáro summation announced as an open problem in [3] was solved by Fridli [1]. Applications of the Orlicz space which is a generalization of $L_p, 1 \leq p \leq \infty$, has been studied extensively, see for examples [4]. The main objective of our (myself, A.H. Siddiqi and S. Fridli) study is to extend the results by Móricz and Siddiqi [3] to the setting of the Besov-Orlicz space. A typic result of our investigation is the following theorem.

Theorem. Let (φ, Ω) be $\Delta-$ regular with complementary $N-$ functions φ . Let $\varphi \in L_\varphi(\Omega)$; let $n = 2^m + k, 1 \leq k \leq 2^m, m \geq 1$, and let $\{q_k : k \geq 0\}$ be a sequence of non-negative numbers satisfying $\frac{n^{\gamma-1}}{Q_n^\gamma} \sum\limits_{k=0}^{n-1} q_k^\gamma = O(1)$ for some $1 < \gamma \leq 2$. Then, for $\{q_k\}$ non decreasing, we have

$$\|\tau_n(f) - f\|_\varphi \leq \frac{5}{2Q_n} \sum_{j=0}^{m-1} (Q_{n-2}\omega_\varphi(f, 2^{-j}) + O\{\omega_\varphi(f, 2^{-m})\} \qquad (1)$$

and for $\{q_k\}$ nonincreasing we have

$$\|\tau_n(f) - f\|_\varphi \leq \frac{5}{2Q_n} \sum_{j=0}^{m-1} (Q_{n-2^j+1} - Q_{n-2^{j+1}+1})\omega_\varphi(f, 2^{-j}) + O\{\omega_\varphi(f, 2^{-m})\} \qquad (2)$$

where $\tau_n(f)$ denotes the n^{th} Nörlund mean of the Walsh-Fourier series of f.

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The radial limit of the hyperbolic derivative and its geometric meaning

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Let φ be a self-map of the unit disk (which will be denoted by Δ). The hyperbolic derivative of φ is defined by

$$\varphi^*(z) = \varphi'(z) \cdot \frac{1-|z|^2}{1-|\varphi(z)|^2}, \quad z \in \Delta\,.$$

If we consider the family of linear maps $\varphi_s : \Delta \to \Delta$ given by $\varphi_s(z) = sz + (1-s),\ 0 < s \leq 1$, it is very easy to obtain that

$$\lim_{r \to 1^-} |\varphi_s^*(r)| = 1\,.$$

The domain $\varphi_s(\Delta)$ is a disk centered at $z = 1-s$ and of radius s. Therefore, all these maps have a corner of opening π at $\varphi_s(1) = 1$.

Let us now deal with the family of lens-maps $L_\alpha : \Delta \to \Delta\ , 0 < \alpha < 1$, defined by

$$L_\alpha(z) = \frac{(\frac{1+z}{1-z})^\alpha - 1}{(\frac{1+z}{1-z})^\alpha + 1}\,.$$

All these maps transform the unit disk onto a domain with a corner of opening $\alpha\pi$ at $L_\alpha(1) = 1$. For this case, we get

$$\lim_{r \to 1^-} |L_\alpha^*(r)| = \alpha\,.$$

Motivated by the examples given above, we consider smooth enough univalent self-maps φ of the disk onto domains with a corner of opening $\alpha\pi$ at a point $\varphi(\zeta)$ of modulus one, and we relate the radial limit of $|\varphi^*|$ at ζ with the opening $\alpha\pi$. We will use classical theorems of Lindelöf and Warschawski and some properties of the hyperbolic derivative such as the subordination principle.

This work is part of the Ph. D. Thesis of the author defended at the Universidad Autónoma de Madrid and directed by Professor D. Vukotić.

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Translational tiles, spectral sets, complex Hadamard matrices

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An open set $\Omega \subset \mathbb{R}^d$ of finite Lebesgue measure is said to be a (translational) *tile* if some disjoint family of translates $\{t+\Omega : t \in \Sigma\}$ of Ω covers $\mathbb{R}^d$(ignoring gaps of measure zero). A set $\Lambda \subset \mathbb{R}^d$ is said to be a *spectrum* of Ω if the functions $\{\frac{1}{|\Omega|^{1/2}} e^{2\pi i \langle x,\xi \rangle}\}_{\xi \in \Lambda}$ form an orthonormal basis of $L^2(\Omega)$ (here $|\Omega|$ denotes the Lebesgue measure of Ω). If such Λ exists, Ω is said to be *spectral.* The spectral set conjecture of Fuglede was formulated 30 years ago, and relates the classes of translational tiles and spectral sets in $\mathbb{R}^d$.

Conjecture An open set $\Omega \subset \mathbb{R}^d$ of finite Lebesgue measure is a tile if and only if it is spectral.

Despite several partial results supporting the conjecture, in 2003 T. Tao [5] showed an example of a spectral set in $\mathbb{R}^5$ which is not a tile. The 'tile → spectral' direction, however, could not be tackled by Tao's arguments. A counterexample for this direction was found, also in dimension 5, in a joint work with M.N. Kolountzakis in 2004, [2]. This talk is centred around the construction of these counterexamples. The main tool being used is discrete Fourier analysis.

Since then, the dimension of the counterexamples have been reduced to 3 in both directions, by works of M. N. Kolountzakis, Sz. Revesz, B. Farkas, P. Mora, and M.M, [3], [1]. At present the Conjecture is still open in dimensions 1 and 2. In dimension 1 the 'tile → spectral' part would follow from a number theoretic conjecture of Coven and Meyerowitz (which is, however, far from being settled...)

As offsprings of the ideas used in disproving Fuglede's conjecture I will also present a tiling of $\mathbb{Z}^2$ possessing *no period vectors* (Kolountzakis, M.M., 2005), and a new *construction of complex Hadamard matrices* [4] (a class of matrices which have raised recent interest due to their use in quantum-information). Also, a similar tiling construction has recently been used by J. Steinberger to produce tilings of $\mathbb{Z}$ with *long periods* (superpolynomial w.r.t. the diameter of the tile; this complements earlier upper bounds on the period by I. Ruzsa, A. Biro).

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Construction of Mexican Hat Wavelet on the Heisenberg Group

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We construct the "Mexican Hat" wavelet on the Heisenberg group, and we shall then look at the class of compactly supported smooth continuous wavelets, with arbitrarily many vanishing moments, on the Heisenberg Lie group.

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On a generalized Shannon approximation– a sharp error estimate and its numerical applications

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The Shannon sampling theorem says that every bandlimited signal can be perfectly recovered from its discrete samples, provided that the sampling period is sufficiently small. However, the Shannon sampling function ($\sin \pi x/\pi x$) does not have compact support(does not even decay sufficiently fast).

We have constructed natural generalized sampling functions φ [5] with compact support[or with fast decay] which yield a good sampling approximation to smooth functions f without bandlimited condition. In fact, these provide actually a simultaneous approximation as well as an exact interpolation formula [2].

A sharp error estimate is established inspired by a beautiful result in [3] on the Coifman scaling function.

To be concrete, our inequality goes as follows: Let φ satisfy the Strang-Fix condition and the moment condition of a certain degree and let the sampling approximation to f be given by

$$S_j f(x) := \sum_{k=-\infty}^{\infty} f(2^{-j}k)\,\varphi(2^j x - k)$$

We assume further that $\varphi(x)$ decays sufficiently fast as $|x|$ goes to infinity. Then, we can prove the following.

$$\| S_j(f) - f \|_{L^p} \quad \leq \quad 2^{-jN} C_{\varphi,p,N} \| f^{(N)} \|_{L^p}, \ \ j = 1, 2, ...$$

Among various applications, we present one to numerical computation of solutions of evolution equations with nonlinear terms [4].

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Hermite expansions of generalized Gelfand-Shilov spaces

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By characterizing Hermite expansions of elements of the generalized Gelfand-Shilov spaces $\mathcal{S}^{(M_p)}{}'$, we give an unifying approach to the investigation of the Gelfand-Shilov spaces $\mathcal{S}_\alpha^\alpha{}'$, Fourier hyperfunction space $\mathcal{F}'$ and spaces of tempered ultradistributions of Roumieu-Komatsu type, which are all special cases of the space $\mathcal{S}^{(M_p)}{}'$, considered in the talk.

We prove that the test space $\mathcal{S}^{(M_p)}$ (in quasianalytic and nonquasianalytic case), can be identified with the space of sequences of ultrafast falloff and its dual space $\mathcal{S}^{(M_p)}{}'$ with the space of sequences of ultrafast growth. Where ultrafast growth means superexponetial or exponential or subexponetnial depending of the sequence M_p, which satisfy conditions of logaritmic convexity (M.1) , stability under ultradifferential operators (M.2) and $p!^{1/2} \subseteq M_p$ (for precise definitions of the conditions see [7]and for the space [1], [6]).

As an application of the characterizations we give a simple proof of the kernel theorem for the spaces $\mathcal{S}^{(M_p)}{}'$. The generalized Gelfand-Shilov spaces are good spaces for Harmonic analysis. The Fourier transform is an isomorphism on them. Therefore from the kernel theorem it follows that the Weyl transform can be extended on generalized Gelfand-Shilov spaces $\mathcal{S}^{(M_p)}{}'$ and that the Pseudodifferential operators can be investigated on more general class of function spaces than in [8].

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On meromorphic functions without Julia directions

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In 1919 G. Julia [3] essentially obtain the following: Let $f(z)$ be a function in the complex plane $\mathbb{C}$. Let z_0 be an essential singularity of $f(z)$. Then there exists at least a ray $\Delta(\theta) = \{z_0 + re^{i\theta} : r \geq 0\}$, where $0 \leq \theta < 2\pi$, emanating from z_0, such that in every angular region $\theta - \varepsilon < \arg(z - z_0) < \theta + \varepsilon$, $\varepsilon > 0$, $f(z)$ assumes, infinitely often, every extended complex value with at most two exceptions.

The ray above is called *a Julia direction for* $f(z)$. Every transcendental entire function f has ∞ as an essential singularity, hence by above Julia's result, we deduce that $f(z)$ has a Julia direction. The *logarithmic order* [2] *of a meromorphic function* f is a number $\limsup\limits_{r\to+\infty} \dfrac{\log T(r,f)}{\log\log r}$, where $T(r,f)$ is the Nevanlinna characteristic function of f. For $\lambda > 2$, any meromorphic function with logarithmic order λ must have a Julia direction, since in [1] it is proved that if f is a meromorphic function with finite logarithmic order, and satisfies the growth condition

$$\limsup_{r\to+\infty} \frac{T(r,f)}{(\log r)^2} = +\infty,$$

then f has a Borel direction with logarithmic order $\lambda - 1$, and a Borel direction of f must be a Julia direction of f. In 1926 A.Ostrowski [4] constructed a meromorphic function f with logarithmic order 2, such that f has no Julia directions.

In this short communication I will present that for any positive number λ, $1 < \lambda < 2$; there exists a meromorphic function f with logarithmic order λ such that f has no Julia directions.

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A survey on Fueter's Theorem in Clifford analysis

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Assume f to be holomorphic in an open set of the upper half of the complex plane and put $f(z) = u(x,y)+iv(x,y)$ $(z = x+iy)$, where as usual $u = \mathrm{Re}\, f$, $v = \mathrm{Im}\, f$. Then Fueter's theorem (see [3]) asserts that in the corresponding region the function

$$\Delta\Big(u(q_0,|\underline{q}|) + \frac{\underline{q}}{|\underline{q}|}v(q_0,|\underline{q}|)\Big)$$

where $\underline{q} = q_1 i + q_2 j + q_3 k$ is a pure quaternion and $\Delta = \partial^2_{q_0} + \partial^2_{q_1} + \partial^2_{q_2} + \partial^2_{q_3}$ the Laplace operator in four dimensional space, is monogenic w.r.t. the quaternionic Cauchy-Riemann operator $D = \partial_{q_0} + i\partial_{q_1} + j\partial_{q_2} + k\partial_{q_3}$.

Fueter's theorem is further generalized in a Clifford analysis setting (see e.g. [4, 5]). In this communication we will discuss a new result, which contains previous generalizations as special cases.

Our methods are an extension of our recent paper "An alternative proof of Fueter's theorem" in cooperation with Qian Tao. For an introduction to Clifford analysis we refer to [1, 2].

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Generalized Halphen-Lamé functions

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As remarked by Painlevé, the problem of determining new transcendental functions defined by ordinary differential equations is one that can be traced back to the work of Abel and Jacobi. The study of equations with solutions having regular behaviour was a topic of great interest during the nineteenth century, with important contributions from Briot and Bouquet, Lamé, Halphen, and Picard, amongst others. This led to the discovery by Painlevé of six equations whose solutions defined new transcendental functions. The study of these equations is of great current mathematical interest [1], as is the discovery of higher order analogues [2].

Halphen [3] studied a remarkable sequence of linear equations, generalizations of a certain Lamé equation. In [4] we consider further generalizations where, instead of the Weierstrass elliptic function, the coefficients depend on the first Painlevé transcendent. We thus obtain generalizations of functions of Halphen-Lamé type, as well as new higher order systems with solutions having only fixed critical points. We also consider the representation of these solutions as power series (obtained from the differential equations), and provide an interpretation of the pattern of conditions for single-valuedness [5] in terms of the domains of convergence of these power series.

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Regularity of the Hardy-Littlewood maximal function

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The study of regularity properties of the Hardy-Littlewood maximal function was initiated by Juha Kinnunen in [2], where it was shown that the centered maximal operator is bounded on the Sobolev spaces $W^{1,p}(\mathbb{R}^d)$ for $1 < p \leq \infty$. Since then, many authors have made different contributions on the regularity of the maximal function: Buckley, Hajłasz, Korry, Lindqvist, Luiro, Onninen, Saksman…

As usual, the case $p = 1$ is essentially different from the case $p > 1$, not only because $L^1(\mathbb{R}^d)$ is not reflexive (so weak compactness arguments used when $1 < p < \infty$ are not available for $p = 1$), but more specifically to this problem because $Mf \notin L^1(\mathbb{R}^d)$ whenever f is nontrivial, while the maximal operator acts boundely on L^p for $p > 1$. Nevertheless, in dimension $d = 1$, Tanaka proved [3] that if $f \in W^{1,1}(\mathbb{R})$, then the noncentered maximal function Mf is differentiable a.e. and $\|DMf\|_1 \leq 2\|Df\|_1$.

In this communication will be presented (mostly) the case $d = 1$ and $p = 1$. What had not been previously noticed, and we show in [1], is that the maximal operator can actually improve the regularity of a function, rather than simply preserving it, and without increasing the variation.

We prove the following. Let I be an interval, let $f : I \to \mathbb{R}$ be of bounded variation, and let Df be its distributional derivative (it is a Radon measure). Denoting by Mf the noncentered maximal function of f we prove that Mf is absolutely continuous and its distributional derivative DMf is a function that satisfies the sharp inequality $\|DMf\|_1 \leq |Df|(I)$.

These results allow us to obtain, under less smoothness, versions of classical inequalities involving a function and its derivatives.

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Topological tensor product and extension groups

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In this paper we first develop the notion of topological tensor products for topological semigroups and we will show that topological tensor product of topological groups S and T is an extension of S and T. Also, we study the universal P-compactifications of this structure.

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Analysis of modified Kontorovitch-Lebedev integral transforms

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A proof of inversion formulas of the modified Kontorovitch-Lebedev integral transforms is developed [1]. The Parceval equations for modified Kontorovitch-Lebedev integral transforms are proved and sufficient conditions for them are found. Some new representations and properties of these transforms are justified. The inequalities which give estimations for their kernels - the real and imaginary parts of the modified Bessel functions of the second kind $ReK_{1/2+i\tau}(x)$ and $ImK_{1/2+i\tau}(x)$ for all values of the variables x and τ are obtained.

The applications of Kontorovitch-Lebedev transforms to the solution of some mixed boundary value problems in the wedge domains are accomplished [2]. The numerical aspects of using of these transforms are elaborated in detail [3, 4].

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On the Aleksandrov Problem for isometric mappings

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Under what conditions is a mapping of a metric space onto itself preserving unit distance an isometry? We discuss some new results as well as pose new related open problems and conjectures [1]. The connection with the Mazur-Ulam theorem for isometric mappings will also be considered.

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Weights for singular integrals with nonstandard kernels

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This is a joint work with M. Lorente, J.M. Martell and C. Pérez. In [1] and [2] it was considered a general framework to deal with singular integral operators and commutators of singular integral operators with BMO functions where Hörmander type conditions associated with Young functions are assumed on the kernels. More precisely, the kernels are those that satisfy the following definition

Definition. Let $K \in L^1_{\text{loc}}(\mathbb{R}^n \setminus \{0\})$, and let $\mathcal{A}$ be a Young function. We say that the kernel K satisfies the $L^{\mathcal{A}}$-Hörmander condition if there are positive numbers $c_{\mathcal{A}} \geq 1$ and $C_{\mathcal{A}}$ such that for any x and $R > c_{\mathcal{A}}|x|$,

$$(H_{\mathcal{A}}) \qquad \sum_{m=1}^{\infty} (2^m R)^n \left\| (K(x-\cdot) - K(-\cdot))\, \chi_{S_{m,R}}(\cdot) \right\|_{\mathcal{A},\, B_{m,R}} \leq C_{\mathcal{A}}$$

We say that a kernel K satisfies the L^{∞}-Hörmander condition if there exist C and $c \geq 1$, such that, for any x and $R > c|x|$,

$$(H_{\infty}) \qquad \sum_{m=1}^{\infty} (2^m R)^n \sup_{2^m R \leq |y| < 2^{m+1} R} |K(x-y) - K(-y)| \leq C.$$

In these cited articles are obtained Coifman type estimates, weighted norm inequalities and two-weight estimates.

In the present work we give weighted weak-type estimates for pairs of weights (w, Sw) where w is an arbitrary nonnegative function and S is a maximal operator depending on the smoothness of the kernel. One-sided singular integrals, as the differential transform operator, are under study. We also provide applications to Fourier multipliers and homogeneous singular integrals.

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Heil-Ramanathan-Topiwala Conjecture for wave packets

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2000 Mathematics Subject Classification. 42C40, 42C15

Wavelet and Gabor analysis on flat spaces (Rand R^n) have attracted the attention of scientists through out the world; see for example [1]. For Extension of these concepts and related results to nonflat structure, say unit sphere, we cite [5]. A system encompassing both wavelet and Gabor system called wave packet system, has been studied, see for example [4]. Heil, Ramanathan, Topiwala [3] explored the basic structure of Gabor frames which ultimately led to a conjecture today known as the HRT conjecture. It states that if $g \in L_2(R)$ is non-zero and $\{(\alpha_k, \beta_k)\}_{k=1}^{N}$ is any set of finitely many distinct points in R^2, then $\left\{e^{2\pi i \beta_k x} g(x-\alpha_k)\right\}_{k=1}^{N}$ is a linearly independent set of functions in $L_2(R)$. Despite the striking simplicity of the statement of this conjecture, it remains open today in the generality stated. A survey paper by Heil [2] provides an updated account of this conjecture. Given $g \in L_2(R)$ and a subset $\wedge \subset R \times R^+$, the collection of the type

$$\{D_b(M_{\beta_k} T_{\alpha_k} g)(t)\} = \left\{b^{1/2} e^{2\pi i \beta_k b t} g(bt-\alpha_k)\right\}$$

is called the wave packet system. Various aspects of this system have been studied by several authors including Kalisa, Torrésani, Labate, Hernández, Siddiqi, Ahmad. However, the HRT conjecture has not been examined for wave packet system. It is an interesting problem because the HRT conjecture for the Gabor system is open but its analogue for wavelet system fails [2]. In this presentation the HRT conjecture for the wave packet system and Gabor system on unit sphere is discussed.

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On entire functions with non-negative Taylor coefficients

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If f is an arbitrary transcendental entire function of finite order and with non-negative Taylor coefficients, we shall prove that $f^* = f/f'$ is a Beurling slowly varying function. We also give a Hardy-type asymptotic equivalence

$$f^{(n)}(r) \sim f(r)/(f^*(r))^n, n \in N(r \to \infty).$$

As an application to the Probability Theory we show that the law

$$F(x) = 1 - 1/f(x), x > 0,$$

is in the domain of attraction of the double exponential law with explicitly given norming constants.

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Multivariate wavelet frames with vanishing moments

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We study construction of wavelet frames with matrix dilation. It is well known that the number of vanishing moments for a wavelet basis depends only on a generating dual refinable function. Situation is essentially different for frames. Two pairs of dual wavelet frames may be generated by the same refinable functions and have different numbers of vanishing moments. We found necessary conditions and a sufficient condition under which a given pair of refinable functions generates dual wavelet systems (potential frames) with vanishing moments up to order n. These conditions are given in terms of polyphase functions. Algorithms for finding compactly supported dual and tight wavelet frames with arbitrary number of vanishing moments are described.

Generalized integrals for vector-latice-valued and Banach-space-valued functions

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Non-absolute integrals of different types are investigated and compared. Among them: Kurzweil-Henstock type integrals with respect to abstract derivation bases defined by generalized Riemann sums (see [3]), Denjoy-Lusin type integrals defined descriptively and the Kolmogorov-Henstock variational integral. These integrals play an important role in solving some problem in Harmonic Analysis on locally compact abelian groups (see [5]) The relation between those integrals essentially depends on whether we consider them in application to the real-valued functions or to the Banach-space-valued functions or at last to the functions with values in Riesz spaces (vector lattices)(see [1]).

In the case of real-valued functions all the three types of integral are equivalent (see [2]). In the Banach-space case the descriptively defined Denjoy-Bochner integral is equivalent to the variational integral, but both are strictly included into the Kurzweil-Henstock integral (see [4]). As for the Riesz-space-valued case, we show that the Kurzweil-Henstock integral is equivalent to the variational integral, but descriptively defined integrals are not in general equivalent to them. The equivalence is obtained only if we impose some additional assumption on the involved Riesz spaces.

The problem of recovering, by generalized Fourier formulae, the Riesz-space-valued coefficients of orthogonal series is solved for some classical systems.

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Distances to convex sets: convex w*-closures versus convex $\|\cdot\|$-closures

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If X is a Banach space and $C \subset X^*$ a convex subset, we investigate whether the distances $d(\overline{co}^{w^*}(K), C)$ are controlled by the distances $d(K, C)$ (that is, if $d(\overline{co}^{w^*}(K), C) \leq M d(K, C)$ for some $1 \leq M < \infty$), when K is a weak*-compact subset of X^*. We obtain, for example, that: (i) C has 3-control if C fails to have a copy of the basis of $\ell_1(c)$; (ii) C has 1-control when $C \subset Y \subset X^*$ and Y is a subspace with w*-angelic unit dual ball $B(Y^*)$. An interesting particular case is when we take C inside X and X is considered canonically embedded into its bidual X^{**}. In this case we get 5-control, in general, and 2-control, when $K \cap C$ is w*-dense in C. The above investigation leads us to study when the convex w^*-closure $\overline{co}^{w^*}(K)$ of a w*-compact subset $K \subset X^*$ coincides with its convex $\|\cdot\|$-closure $\overline{co}^{\|\cdot\|}(K)$. We give characterizations of this fact.

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Convergence of trigonometric series

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We consider the series

$$\sum_{n=1}^{\infty} a_n \cos nx \tag{1}$$

and

$$\sum_{n=1}^{\infty} a_n \sin nx, \tag{2}$$

where $\{a_n\}_{n=1}^{\infty}$ is given a null sequence of complex numbers. We define by $f(x)$ and $g(x)$ the sums of the series (1) and (2) respectively at the points where the series converge.

Definition. The sequence of complex numbers $a = \{a_n\}_{n=1}^{\infty}$ is said to be *general monotone*, or $a \in GM$, if the relation

$$\sum_{\nu=n}^{2n-1} |a_\nu - a_{\nu+1}| \le C|a_n|$$

holds for all integer n. We prove the following theorems.

Theorem 1. *Let* $\{a_n\}_{n=1}^{\infty} \in GM$ *and* $\sum\limits_{n=1}^{\infty} \frac{|a_n|}{n} < \infty$. *Then series (1) and (2) converge for all* x, *except possibly* $x = 2\pi k$, $k \in \mathbf{Z}$ *in the case of (1), and converge uniformly on any interval* $[\varepsilon, 2\pi - \varepsilon]$, *where* $0 < \varepsilon < \pi$.

Theorem 2. *Let* $\{a_n\}_{n=1}^{\infty}$ *be a positive sequence and* $a \in GM$. *Then a necessary and sufficient condition for the uniform convergence of series (2) (or for (2) to be a Fourier series of a continuous function) is condition* $\lim\limits_{n\to\infty} na_n = 0$.

Theorem 3. *Let* $\{a_n\}_{n=1}^{\infty} \in GM$ *and let (1) be the Fourier series of a function* $f(x) \in L_1$. *Then* $\lim\limits_{n\to\infty} \|f(\cdot) - S(f,\cdot)\|_1 = 0 \iff \lim\limits_{n\to\infty} \ln n|a_n| = 0$. *The same results hold for series (2) and* $\sum\limits_{n=-\infty}^{\infty} a_n e^{inx}$.

Theorem 4. *Let* $a = \{a_n\}_{n=1}^{\infty}$ *be a positive sequence and* $a \in GM$, *and let* $1 < p < \infty$. *Then* $f(g) \in L_p \iff \sum\limits_{n=1}^{\infty} a_n^p n^{p-2} < \infty$.

These results are partially known: for monotone coefficients [4], quasimonotone [1,3], O-regularly varying quasimonotone [3], RBVS [2].

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Some remarks about Laguerre function expansions

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We give descriptions of power weighted inequalities, of strong, weak and restricted weak type for heat diiffusion semigroup and Riesz transforms associated to several Laguerre function systems.

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Classes of weighted function spaces

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The aim of this work is to show how several important results of $F(p,q,s)$ spaces of analytic functions introduced by Zhao [1] are true in a more general context.This means that some results of this theory does not depend of the analytic properties of the functions so we can obtain similar results by considering more general functions

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Geometric properties of Carnot-Carathéodory spaces and differentiability

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An equiregular *cc*-space $\mathbf{M}$ is characterized as a connected Riemannian manifold with a distinguished horizontal subbundle $\mathbf{M}$ in the tangent bundle $T\mathbf{M}$ that meets some algebraic conditions on the commutators of vector fields $\{X_1, \dots, X_n\}$ constituting a local basis in $H\mathbf{M}$, $n = \dim H\mathbf{M}$. The the intrinsic Carnot–Carathéodory metric $d_{\mathbf{M}}$ between points $x, y \in \mathbf{M}$ is defined as the infimum of the lengths of *horizontal* curves joining x and y (a piecewise smooth curve γ is called horizontal if $\dot{\gamma}(t) \in H_{\gamma(t)}\mathbf{M}$). A tangent cone of a *cc*-space at a point $g \in \mathbf{M}$ in the sense of Gromov [1] is a nilpotent Lie group with a Carnot–Carathéodory metric. It is convenient to regard a neighborhood $\mathcal{G}^g$ of a point g as a subspace of the metric space $(\mathbf{M}, d_{\mathbf{M}})$ and as a neighborhood of unity of the local Carnot group with Carnot–Carathéodory metric d^g.

Given *cc*-spaces $(\mathbf{M}, d_{\mathbf{M}})$ and $(\mathbf{N}, d_{\mathbf{N}})$ and a set $E \subset \mathbf{M}$, a mapping $f : E \to \mathbf{N}$ is called *hc-differentiable* at a point $g \in E$ if there exists a horizontal homomorphism $L : (\mathcal{G}^g, d^g) \to (\mathcal{G}^{f(g)}, d^{f(g)})$ of the nilpotent tangent cones such that $d_{\mathbf{N}}(f(w), L(w)) = o(d_{\mathbf{M}}(g, w))$ as $E \cap \mathcal{G}^g \ni w \to g$. We generalize a result of [2].

Theorem. *Any mapping defined on an arbitrary set in a Carnot–Carathéodory space with values in another Carnot–Carathéodory space and meeting a condition* $\overline{\lim\limits_{y \to x}} \frac{d_{\mathbf{N}}(f(x), f(y))}{d_{\mathbf{M}}(x, y)} < \infty$ *almost everywhere is cc-differentiable almost everywhere.*

The theorem is proved under minimal smoothness of vector fields. Its proof is based on *hc*-differentiability of rectifiable curve on $\mathbf{M}$ [3]. I am going to exhibit some applications of the theorem to geometry of *cc*-spaces, to geometric measure theory, to Sobolev spaces and to quasiconformal analysis. In particular, I will show that the correspondence “local basis ↦ nilpotent tangent cone” has a functorial property, Sobolev mappings between *cc*-spaces are differentiable in a Sobolev topology, quasiconformal mappings between *cc*-spaces are *hs*-differentiable (this is stronger differentiability comparing with Margulis-Mostow paper).

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On a general class of q-polynomials suggested by basic Laguerre polynomials

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Having defined a q-extension of the polynomial $L_n^{\alpha,\beta}(x)$, we investigate its fundamental properties such as q-generating relation, q-partial difference equation and recurrence relations. A generalized q-generating function for the said polynomial is also established. It has further been shown that the newly defined polynomial is closely related to the q-Laguerre polynomial $L_n^{\beta}(x;q)$. Certain interesting limiting cases in the form of the known results due to Prabhakar and Rekha [Math. Student, 40(1972), 311-317] and Prabhakar [Pacific J. Math. 35(1)(1970), 213-219] have also been discussed. Some of the main results proved in this paper are as under:

(a) A q-extension of $L_n^{\alpha,\beta}(x)$:

$$L_n^{\alpha,\beta}(x;q) = \frac{\Gamma_q(\alpha n+\beta+1)}{(q;q)_n}\sum_{j=0}^{n}\frac{(q^{-n};q)_j\ (xq^n)^j\ q^{j(j-1)/2}}{(q;q)_j\ \Gamma_q(\alpha j+\beta+1)}, \tag{1}$$

where $Re(\alpha) > 0$ and $Re(\beta) > -1$.

(b) q-generating function:

$$\sum_{n=0}^{\infty}\frac{L_n^{\alpha,\beta}(x;q)t^n}{\Gamma_q(\alpha n+\beta+1)} = e_q(t)\phi(\alpha,\beta+1;q,-xt), \tag{2}$$

where $\phi(\alpha,\beta+1;q,-xt)$ is q-Bessel-Maitland function.

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Kolmogorov problem on widths asymptotics

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Given a compact set K in an open set D on a Stein manifold $\Omega, \dim \Omega = n$, the set A_K^D of all restrictions of functions, analytic in D with absolute value bounded by 1, is considered as a compact subset in $C(K)$. The problem about the strict asymptotics for Kolmogorov diameters (widths) of this set:

$$-\ln d_s\left(A_K^D\right) \sim \sigma\ s^{1/n},\ s \to \infty. \tag{1}$$

was stated by Kolmogorov (1956) in an equivalent formulation for ε-entropy of this set. It was conjectured in [3, 4] that for "good" pairs $K \subset D$ the asymptotics (1) holds with $\sigma = 2\pi\left(\frac{n!}{C(K,D)}\right)^{1/n}$, where $C(K,D)$ is the Bedford-Taylor pluricapacity of the "pluricondenser" (K,D). In the one-dimensional case this hypothesis is equivalent to Kolmogorov's conjecture about asymptotics of ε-entropy of the set A_K^D, which has been confirmed by efforts of many authors (Erokhin, Babenko, Zahariuta, Levin-Tikhomirov, Widom, Nguyen, Skiba - Zahariuta, Fisher - Miccheli, et al). In [3, 4] it was described how to reduce the problem (1) in multidimentional case to the problem of Pluripotential Theory about approximation of the relative Green pluripotential of the pluricondenser (K,D) by pluripotentials with finite set of logarithmic singularities (the detailed proof of the reduction is in [5]). The last problem has been solved recently by Nivoche and Poletsky [1, 2], which results, together with [3, 4, 5], a final proof of our conjecture about strong asymptotics (1) under some natural restrictions about K, D. In our talk we discuss the results of [5] including some recent progress in related topics.

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Distributions in Clifford analysis

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In [1] and [2], four broad families of distributions in Euclidean space $T_{\lambda,p}, U_{\lambda,p}$, $V_{\lambda,p}$ and $W_{\lambda,p}$, depending on parameters $\lambda \in \mathbb{C}$ and $p \in \mathbb{N}_0 = \{0, 1, 2, \ldots\}$, were introduced and studied in the framework of Clifford analysis, a function theory centred around the notion of monogenic functions, i.e. null solutions of the Dirac operator $\underline{\partial} = \sum_{j=1}^{m} e_j \, \partial_{x_j}$. These distributions all spring from the already classically known distributions $T_\lambda = \mathrm{Fp}\, r_+^\lambda$. Here Fp is the fundamental distribution "finite parts" on the real line, and spherical coordinates ($\underline{x} = r\underline{\omega}$, $\underline{x} \in \mathbb{R}^m$, $r = |\underline{x}|$, $\underline{\omega} \in S^{m-1}$) have been used to convert an originally m–dimensional distribution into one acting on the real line. More precisely,

$$\langle\, T_\lambda \;,\; \phi \,\rangle \;=\; a_m \,\mathrm{Fp} \int_0^{+\infty} r^{\lambda+m-1} \left[\frac{1}{a_m} \int_{S^{m-1}} \phi(\underline{x}) \, dS(\underline{\omega}) \right] dr = \left\langle\, \mathrm{Fp}\, r_+^{\lambda+m-1} \;,\; \Sigma^{(0)}[\phi] \,\right\rangle$$

where a_m is the area of the unit sphere S^{m-1} in $\mathbb{R}^m$ and $\Sigma^{(0)}[\phi]$ denotes the so-called spherical mean of the testing function ϕ, obtained through integration over the unit sphere. An analogous approach underlies the definition of the four families of distributions mentioned above, which respectively involve the inner spherical monogenics $P_p(\underline{\omega})$ ($p \in \mathbb{N}_0$), i.e. restrictions to the unit sphere of monogenic polynomials which are vector valued and homogeneous of degree p, and the related outer spherical monogenics $P_p(\underline{\omega})\underline{\omega}$, $\underline{\omega}P_p(\underline{\omega})$ and $\underline{\omega}P_p(\underline{\omega})\underline{\omega}$.
In the recent past, we have elaborately studied those four families of distributions. We have investigated a.o. their behaviour in frequency space (see [3]) and the definition and properties of both the convolution and the product of arbitrary elements of the families of distributions under consideration, leading to a very attractive pattern of mutual relations between them (see [4]). In this talk we would like to present an overview of our results.

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Uniform and pointwise convergence of q-Fourier-Bessel series

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We define q-analogues of Fourier Bessel series, by means of complete q-orthogonal systems constructed with the third Jackson q-Bessel function

$$J_\nu^{(3)}(z;q) = z^\nu \frac{(q^{\nu+1};q)_\infty}{(q;q)_\infty} \sum_{k=0}^{\infty} (-1)^k \frac{q^{\frac{k(k+1)}{2}}}{(q^{\nu+1};q)_k (q;q)_k} z^{2k}, \tag{1}$$

where $\nu > -1$ is a real parameter, i.e., series of the form:

$$S_q^\nu[f](x) = \sum_{k=1}^{\infty} a_k\,(f)\, J_\nu(q j_{k\nu} x; q^2), \tag{2}$$

Sufficient conditions for pointwise and uniform convergence of these series are obtained. The convergence results are illustrated with some examples of developments in q-Fourier Bessel series.

These convergence results together with the properties of the zeros of the third q-Bessel function [3, 4], the completeness properties [2], a q-integral Hankel transform [4] with an inversion formula for the whole grid $q^k, k = 0, \pm 1, \pm 2, ...$, and the associated reproducing kernel Hilbert space [1]) lead to a very complete q-Fourier series theory.

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Weighted almost periodicity

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Let G be a locally compact abelian group and ω be a weight function on G, this study is concerned with the weighted almost periodic elements of G-modules and $L^1_\omega(G)$ modules and the relationship between the various definitions.

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