

**ICM 2006**

**Posters**

**Abstracts**

**Section 19**

**Mathematics Education and  
Popularization of Mathematics**

## Is there mathematics in The Quixote?

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In The Quixote [1], a nobleman of La Mancha, driven mad by chivalresque readings, believes himself to be a knight-errant and leaves his village in search of adventures, always really silly things, in a principal scene: La Mancha, and its people, carriers of universal realities. There is a link between the concrete thing and its universal significance that makes this novel be deeply local and universal. Mathematics contains abstract components that become general through concrete realities. Mathematical education tries to build a cognitive bridge that links the concrete reality with the ideal posed by the problem to be solved. So, in this poster we propose a collection of mathematical activities related to The Quixote [2] and presented with these epigraphs:

- Itineraries: In this epigraph we connect Euler's and Hamilton's graphs with Don Quixote's travels. The adventure of the windmills, where we propose a classic mathematical game based on the destruction of the windmills by our protagonist.
- Numbers in The Quixote: The strange a million six hundred thousand. We show a description of the numbers appearing in the novel.
- Crossing the river: A paragraph from the novel is a pretext to propose classic logic problems.
- Cryptograms: We present some cryptograms with typical words of Cervantes's novel.
- The Quixote paradox: We show a paradox implicit in the novel. It is an example of the many paradoxes present in mathematics from its origins.

## References

- [1] Cervantes, M., El ingenioso hidalgo Don Quijote de la Mancha, Espasa Calpe S.A.

- [2] Carlavilla, J.L., Si hay una X hay matemáticas, Proyecto Dur de Ediciones S.L., 2005.

## CIMM: interactive course of mathematics with Maple

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We are presenting here an interactive electronic book of introductory Mathematics with Maple, the CIMM (Curso Interactivo de Matematicas con Maple). This e-book has been developed in the ICE (Instituto de Ciencias de la Educacion), by several teachers of the UPM (Universidad Politecnica de Madrid), as a self-teaching aid for 1st year students with poor mathematical knowledge. It complies with the EEES (Espacio Europeo de Educacion Superior) directions on new methodologies based on Information and Communications Technologies.

It has been structured to allow random access to the contents, facilitating quick and easy retrieval of basic concepts, examples, etc. Interactivity has been a key principle for the authors: students can experiment with different inputs, to improve understanding. A fundamental feature of the course is intensive use of hypertext, which facilitates links to review topics according to the individual needs of the student. The CIMM [1] is free and only requires MAPLE software, which is available to all students in the UPM.

Contents of the course include Linear Algebra [3] (vectors, matrices, linear systems) Analysis [2] (elemental functions, limits, differential and integral calculus) and Geometry (conics). All of them treated with enough detail to satisfy the mathematical prerequisites for scientific and engineering university studies.

Besides the fundamental theory, this e-book is mainly based on a complete collection of exercises, some of them "real life" applications. Plenty of graphics and animations have also been included throughout the text to enhance intuitive understanding.

## References

- [1] <http://www.eui.upm.es/%7Egarcial/>

- [2] Larson, R., Hostetler, R.P., Edwards, B.H. *Calculo I y II*. Ed. Piramide. 2003.
- [3] Lay, D.C. *Algebra Lineal y sus Aplicaciones*. Ed. Pearson. 1999.

## ESTALMAT: A project for selecting and nurturing talented children aged 13 to 15

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Studies indicate that formal reasoning starts around 12-13 years of age. Guided by these studies, in 1998 Professor Miguel de Guzmán (1936 - 2004) designed a project to select each year 25 mathematically gifted children in the region of Madrid (Spain). These children take part in a two-year program devised to nurture their talent through the enhancement of their reasoning skills, as opposed to computational skills. Classroom time is based in teamwork and collaboration among students with the guidance of experienced mathematical teachers.

With the organizational support of the Spanish “Real Academia de Ciencias Exactas, Físicas y Naturales”, and the financial support of the “Fundación Vodafone España”, the project has been successfully extended to Cataluña, Andalucía, Castilla y León and Canarias, and about 100 talented young people start the program every year.

The poster presentation will address the following aspects of the program:

- The selection process, including examples of exercises used in previous years.
- The structure and organization of the program.
- The type of mathematical activities designed to use with talented children.
- Results of studies done over the years with the children selected.

The presentation is done on behalf of all the mathematics teachers that are currently working on the program.

## References

- [1] *[www.uam.es/estalmat](http://www.uam.es/estalmat)*

## The self-knowledge applied to the study

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I present an educational experience for students that has been designed in a few sessions of self-knowledge and auto-learning. It has been developed along a four-month period with the intention and purpose that the student feels more alive the experience and has the opportunity of knowing oneself, not through received information, or by intellect, but through his very being, through his own core structure. In each session the student, guided by the teacher, thinks about, investigates and experiments in relation to his learning process, maturation process in the subject and personal process of being student. Another objective is that each student follow a process of personal alive and of conscious work, which provides him of helpful skills for self-knowledge and growth as student and as person making possible to being.

What it motivates to me is the desire from which the student becomes aware of his own experience and of itself in his time of student, and my desire that he learns by means of the discovery, that is, through the experience, in order to understand and to know. According to my own experience, the possibility of discovering is closely related with the experience, and is in this sense that I emphasize in the need of being the process alive.

My firmness and faith in the project, the conviction of the usefulness of the sessions for the student's personal growth, the confidence in me, and the importance that I give to the attitude, the disposition and the experience, allow me to performance it safely and efficiently. Which push me to make it is, essentially, my confidence that the teacher's *attitude* and *presence*, his transparency, his capacity of catching, his own self-knowledge process and personal growth, influence the learning process of the student.

## References

- [1] Naranjo, Claudio, *Cambiar la Educaci3n para cambiar el mundo*. Ediciones La Llave, D.H. 2004. (*To change the Education to change the world.*)
- [2] Perls, Fritz, *El enfoque Gestáltico*. Editorial Cuatro Vientos, 2000. (*The Gestalt Approach*. Science and Behavior Books, 1973.)

- [3] Rogers, Carl, *El proceso de convertirse en persona*. Editorial Paidós, 1977. (*On Becoming a Person*. Boston, Houghton Mifflin, 1961.)
- [4] Stevens, John O., *El Darse Cuenta. Sentir, imaginar y vivenciar*. Editorial Cuatro Vientos, 2003. (*Awareness: exploring, experimenting, experiencing*. Real People Press, 1971.)
- [5] Zinker, Joseph, *El proceso creativo en la Terapia Gestáltica*. Editorial Paidós, 2000. (*Creative Process in Gestalt Therapy*. Brunner/Mazel, Inc, New York, 1977.)

## DIDIMA: dialogues on discrete mathematics

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DIDIMA is a project aiming to establish the documental framework and guidelines for future research projects concerning discrete mathematics and education. This includes: a) identifying and assessing activities that fit within the scope of all stakeholders involved, as well as a careful review of existing educational practices in discrete mathematics [2], and b) continuously monitoring, reviewing and evaluating the progress and performance of the project. DIDIMA is a stimulus for the creation of structural links and networks in mathematics and science education within the European Research Area, whose objectives are likely to be stated from a dynamic dialogue between scientists, educators, policy makers and the society at large. It intends to impulse and catalyse European projection and cooperation by contributing to educational standards [1], multidisciplinary education networks, and policy development. These levels of contributions will have a concrete impact by:

- Increasing the interest of the community of science and mathematics educators in the field of discrete mathematics, enabling them to recognize it as being not just another particular curricular topic, but instead a valuable tool for linking science and technology.
- Increasing the number of research articles, particular actions, and essays with the joint effort of science and mathematics education.
- Consolidating a research group in discrete mathematics and education, open to other disciplines and well informed about social requirements in this field.

## References

- [1] Lutz-Westphal, B., Erlebnis Mathematik - Kombinatorische Optimierung im Unterricht, *Mitteilungen der Deutschen Mathematiker-Vereinigung* **2** (2004), 78–81.

- [2] Rosenstein, Joseph G., Ed.; Franzblau, Deborah S., Ed.; Roberts, Fred S., Ed., *Discrete Mathematics in the Schools*, DIMACS Series in Discrete Mathematics and Theoretical Computer Science, Volume 36, Providence, RI: American Mathematical Society (AMS), 1997.

**Analysis of a constructivist experiment with ICT in mathematics education**

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The use of information and communication technologies (ICT) in society is increasing at a phenomenal rate; however, their daily application in schools is much more limited [2]. Even in schools that have adequate equipment and technologically trained staff, the use of these facilities is neither widespread nor continuous. This reticence when it comes to using these new resources might be the result of general uncertainty about the academic results gained with them. In order to try and assuage these doubts, this poster presents the results obtained in a mathematics education experiment using ICT and a constructivist methodology [1], [3]. This experiment was conducted as part of a project [4] promoted by the Spanish Ministry of Education in twenty six secondary schools.

Statistical analysis of the results reveals that the students who participated in this experiment experienced a statistically significant improvement in their learning in comparison to students from the control groups. This improvement was not only observed among students with a previously acceptable and good global academic performance, but also in students who had been failing at school. The students felt that the experience was very positive that their learning had been optimized; they expressed a desire to continue with these methods and extend them to other subjects.

## References

- [1] Fosnot, C., *Constructivism: Theory, perspectives, and practice*. Teachers College Press, New York, 1996.
- [2] Marchesi, A. y Martín, E., *Tecnología y aprendizaje. Investigación sobre el impacto del ordenador en el aula*. Ed. S.M., Madrid, Spain, 2003. Retrieved from the Web 3/22/06. <http://www.piloto.librosvivos.net/>
- [3] Piaget, J., *The development of thought: Equilibration of cognitive structures*. The Viking Press, New York, 1977.
- [4] Proyecto Descartes, [Educational Project & Computer Software]. Spanish Ministry of Education. Retrieved from the Web 3/22/06. <http://descartes.cnice.mecd.es/ingles/>

## Exploring dynamic movements in geometry with Cabri géomètre

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In this paper I solve a problem in Plane Geometry by exploring the continuity of the movement of the points in the space. Moreover, I analyze how to use Cabri Géomètre to explore these movements in teaching Euclidian Geometry. A nice feature of this software is that it offers the possibility to observe the dynamic movements of geometrical constructions, a task that would otherwise be done step by step if using ruler and compass. More important, this dynamic observation stimulates the mental construction of the visual image in a stronger way than in the step-by-step procedure. Different geometrical problems and conjectures can emerge by letting the student or the instructor imagine how the geometrical figures result out of the movements of points in Cabri. I borrow Vigotsky's ([6]) ideas about how spontaneous and scientific concepts are formed to suggest that the dynamic construction of geometrical concepts and figures with Cabri adds flexibility to the learning process. Moreover, such flexibility also depends on the close participation of the student in moving and constructing geometrical figures in Cabri.

## References

- [1] Coxeter, H. S., *Introduction to Geometry*. John Wiley & Sons Inc., New York, 1969.
- [2] Duarte Jr., G. G., Grass, I. P., La perspectiva histórico-cultural y el aprendizaje de la Matemática, In: *XVI Reunión Latino Americana de Matemática Educativa*, Havana (Cuba), 2002.
- [3] Duarte Jr., G. G., Grass, I. P., Modelo e Realidade no Ensino da Matemática Financeira. In *III CNMEM*, Piracicaba, São Paulo (Brazil), 2003.
- [4] Duarte Jr., G. G., Grass, I. P., Vigotsky: Desenvolvimento e o ensino da Matemática no Terceiro Grau. In *II Jornada do Núcleo de Ensino*, Marília, São Paulo (Brazil), 2003.
- [5] Lebesgue, H. *Measure and the Integral*. HODEN-DAY INC., San Francisco, 1966.
- [6] Vigotsky, L. S. *Obras Escogidas II – Aprendizaje*. Vizer, Madrid, 1995.

## Mathematics in business administration

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This is a study , from different points of view, of Mathematics in Bachelor of Science in Business Administration which can also be applied to other areas of Economics.

It is divided into three parts: first, the study of Mathematics in pre-university secondary level to be used as a tool in this undergraduate university course; second, a similar study of Mathematics in the university level and third, a study of Mathematics used in undergraduate university courses in different Spanish and some European universities. These studies are based on the theory of latent features which makes possible the allocation of Rasch measure to the items considered, which, in this case, are the contents of Mathematics as a subject.

The measures obtained can be intuitively and graphically represented.

Finally, the conclusions will give us information and knowledge about the mathematical concepts necessary in the area of Economics.

Although it is considered within Social Science, an important base in Mathematics is a requirement throughout its study towards the attainment a college degree.

## References

- [1] Heras, A. and others, *Matemáticas Empresariales. Determinación de los contenidos docentes en la licenciatura de Administración y Dirección de Empresas*. Innovación Educativa. Editorial Complutense, Madrid, 2005
- [2] Rasch, G., *Probabilistic Models for Some Intelligence and Attainment Tests*. The University of Chicago Press, Chicago 60637, The University of Chicago, Chicago, 1980

**Application of the new information technologies to the decrease of the failure in the learning of mathematics in biological sciences undergraduate students**

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At present most freshmen students starting their studies in our Center lack of enough background of mathematics exhibiting in many cases a ‘fear’ towards the study of this subject. In this paper we present a discussion about the advantages and disadvantages of two different teaching strategies oriented to motivate the students in order to reduce the failure in the learning of the basic concepts of Mathematics. The study has been carried out with undergraduate students of Biological Sciences degree. One of this approaches, the classical approach, is well known in universities around the world consisting in the hands-out training of students in the computer classroom using a computer program, i.e. DERIVE<sup>TM</sup>, or other software oriented to symbolic calculus with equivalent characteristics. On the other hand, the other approach consists in a multimedia course designed and developed by the authors of this paper. The course is a CD [1] including the basic topics and issues in a course of Mathematics, including Java software and applets such as ‘Plane Graphic Calculator’ and ‘Java Components for Mathematics’. Using the CD the students can update the contents of the CD via Internet (download of free software, news, selected links, etc.), visiting the web site at [2]. In this web site and using Hot Potatoes 6.0 we developed a test with Mathematics exercises and problems evaluating the students their own knowledge, bearing the test a resemblance with Piaget’s style of teaching.

## References

- [1] Martínez Calvo, M. C., Fernández Bermejo, E., González Manteiga, M. T., Lahoz Beltra, R., Perales Graván, C. 2005. Matemáticas Básicas para Biólogos. CD-ROM, Editorial Complutense ISBN 84-7491-786-7.
- [2] <http://www.ucm.es/info/matbio/piembb/addmhb.html>

## Motivational numerical examples for electrical/electronics engineers

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Falling parachutists and diving spheres do not absolutely motivate an undergraduate electrical/electronics engineer to study a first course on numerical methods. On the other hand, we also have to convince our colleagues for the need to include advanced numerical courses in their forthcoming postgraduate engineering programs. Unfortunately, to collect a handful of specific examples to illustrate every aspect of a numerical course is a tedious task, but the outcomes are highly satisfactory.

Apart from several electrical/electronics numerical examples that can be found in usual application-oriented textbooks on numerical methods, we have adapted some examples from electrical/electronics engineering textbooks as those on circuit theory [1], power system analysis [2] and adaptive filter theory [3], as well as from more advanced sources like [4]. The adaptation is by no means trivial: it has to be done in a graphical and direct manner, because each example must be explained in no more than five minutes since we do not want to spend more than two hours in total throughout the course.

This battery could be helpful for other numerical teachers during their lesson preparation to highlight some specific engineering applications that increase the degree of popularization of numerical analysis, in the same way that a magician has her own bouquet of enchanting tricks. These are our twenty-four cards. Which ones are yours?

## References

- [1] L. Chua, C. Desoer y E. Kuh. *Linear and Nonlinear Circuits*. McGraw-Hill, New York, 1987.
- [2] J. Grainger and W. Stevenson. *Power System Analysis*. McGraw-Hill, New York, first edition, 1994.
- [3] S. Haykin. *Adaptive Filter Theory*. Prentice-Hall, Upper Saddle River, New Jersey, third edition, 1996.

- [4] S. Vavasis. Stable numerical algorithms for equilibrium systems. *SIAM J. Matrix Anal. Appl.*, 15(4):1108–1131, October 1994.

## Mathematician waiting for the wedding cake and the cava at the reception

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Imagine you are at a wedding reception. You have been sitting at the table for two hours and the cake, the wine and the coffee still must be brought. You know nobody except your partner. The interesting conversations and the good jokes have already been told, and you don't even remember the name of more than half of the people sitting at your table. Although the guests don't believe it, mathematics has been present during the meeting, and you, of course, are going to tell some stories about mathematics and what you have just eaten. Some of your comparisons probably will leave when you start, but those who stay may have an enjoyable time and may even learn something new. The volume of alcohol ingested by all of you might influence the development of the exposition. Drink in moderation and wait for the dancing and the free bar for to start being yourself

## References

- [1] Grupo LaX, *El truco está en la Geometría*, Investigación en el Aula de matemáticas. Granada, 2005.
- [2] Ramírez-Uclés, R. *Dragones, enanos, jinetes y un divertido mundo al revés*, Revista de la Sociedad Andaluza de Educación Matemática THALES, 2004.

**The relation between the evaluation of primary and secondary school students in mathematics with the student's choice of type of school (day school or evening school) and the role of students' gender**

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In previous years more often the students dropped out the compulsory education because of social factors. Researches have supported this case. In our country, researches (Drettaki 1993) have shown, that factors as the record of students in the various stages of education influence the abandonment of compulsory school. In an OVE research (1996), an important number of students that interrupted the school, declared weakness in the course of mathematics. Others researches have shown (Raty & Snellman, 1995) that the teachers' opinions students' opinion about the students' knowledge in mathematics influences them about their abilities (Leder, Pearn, Brew & Bishop, 1997) for farther education. In our country (Greece) the experience has sown that the "good" students prefer a day than an evening school to keep on their studies.

This presentation is part of a wider study which aimed to investigate social and educational factors that influenced students dropping out of school in the Dodecanese islands of Greece, so that:

- They would abandon or interrupt their studies for a long or sort period after primary school or secondary school
- The relation of dropping out of school to the failure in mathematics in all stages of education.

We studied and will present data from the analysis of 344 questionnaires (204 from gymnasium, 140 from lyceum) which were collected from evening high school in Rhodes.

In our study, we will present the relation between the fore mentioned two factors with:

- the relation between the students' choice about the school type (day or evening high school) and the students' evaluation in mathematics during the primary school.

- the relation between the students' choice about the school type (day or evening senior high school) and the students' evaluation in mathematics during the primary school.
- the relation between the students' choice about the school type (day or evening high school) and the self-rating in mathematics during the primary school
- the relation between the students' choice about the school type (day or evening senior high school) and self-rating in mathematics during the primary school
- The role of their gender for those choices.

## References

- [1] Drettaki, M., *The abandon of the obligatory education and the factors that relate with it*. Grigoris, Athens (in Greek), 1993.
- [2] OVE (Organization of Vocational Education) - Pedagogic Institute, *The students who abandon their studies in high school and their needs for Vocational Education*, Athens (in Greek), 1996
- [3] Raty, H., Snellman, L., On the social fabric of intelligence. In *Paper on Social Representations*, 4, 177-185, 1995
- [4] Leder, G. C., Pearn, C., Brew, C., Bishop, A. J., *I'd like to be really good at mathematics*, Publisher: Internet, 1997

## Reinventing the wheel: non-circular wheels

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We set to prove the impossibility of non-circular wheels, and surprised ourselves by proving the opposite. When included as components of a vehicle, wheels allow the vehicle to roll smoothly over a surface. An axle placed at the centre of a circular wheel will stay at a constant altitude from the ground as the wheel rotates. An early approach to using non-circular wheels-albeit not a practical one-consisted in modifying the shape of the surface to prevent the up-and-down motion of the centre of gravity of the non-circular wheel.

Here we present a different solution: non-circular figures can be used as wheels not rollers, yet allow the vehicle to run smoothly on a level surface, by modifying the shape of the axle! We present constant-width wheels, triangular-based wheels, general non-circular wheels and what we call "squared" and "triangular" wheels.

Ironically, this idea was the result of an unsuccessful attempt to prove the following assertion from a fascinating book, *The Enjoyment of Mathematics* [3]:

*Obviously a wheel must be made in the form of a circle with the hub at the centre, since any other form will produce an up-and-down motion.*

## References

- [1] Gardner, M., *Mathematical games: Curves of constant width, one of which makes it possible to drill square holes*. In *Scientific American*, 78:494, (1963).
- [2] Montejano, L., *Cuerpos de Ancho Constante*, Fondo de Cultura Económica, Mexico, 1998.
- [3] Rademacher, H., Toeplitz, O., *The Enjoyment of Mathematics*. Princeton University Press, Princeton, NJ, 1957.
- [4] Santaló, L. *Integral Geometry and Geometric Probability*. Addison- Wesley, Reading, 1976.
- [5] I. M. Yaglom, I. M., Boltyanskii, V. G., *Convex Figures*, Holt, Rinehart and Winston, New York, 1961.

**The influence of teaching the concept of function using graphing calculators. A venezuelan experience**

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Function is the single most important concept from kindergarten to graduate school and is critical throughout the full range of education. The introduction of functions and graphs is a critical point in mathematics education because these topics are fundamental to more sophisticated study of mathematics. In the Principles and Standards for School Mathematics (2000), the National Council of Teachers of Mathematics recommend that, by the ninth grade, students will have represented linear functions with tables, graphs, verbal rules, and symbolic rules and worked with and interpreted these representations. Hollar (1996) examined the effects of a graphing-approach college algebra curriculum along with the TI-82 graphing calculator on students' understanding of the function concept. Graphs are a widely used way to represent functions, especially from the real numbers to the real numbers. If we can draw the graph of a function we can talk about some characteristic of the functions such as domain, range, maxima, minima, concavities, etc. Students should use the technological tools to represent and study the behavior of polynomial, exponential, rational, and periodic functions, among others. With access to graphing calculators that completes manipulations of symbolic expressions or equations, students can compute or approximate values of functions or solutions to equations, and can graph functions and relations. Thus, students can easily explore the effects of changes in a parameter as a means of better understanding classes of functions. It is the goal of the poster to show how graphing calculator (Voyage 200) can help in the teaching and learning of mathematics process the concept of function

## References

- [1] Hollar, J., *The effects of a graphing approach college algebra curriculum on a students' understanding of the function concept*. Doctoral Dissertation, North Carolina State University, Dissertation Abstracts International, 57, 07A, 1996.
- [2] National Council of Teachers of Mathematics, *Principals and standards for school mathematics*. Reston, VA: The Council, 2000.

## Linear algebra on demand

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Teaching mathematics is undergoing dramatic changes since computers are becoming part of our daily life. Starting from school, students have less and less motivation to perform calculations ‘by hand’. By contrast, there is an increasing demand to understand what is behind algorithms. Mathematicians may consider this development an advantage, giving evidence of increasing importance of our discipline as a ‘key technology’ in society.

But is it really necessary – say, for an engineer – to understand subtle questions of pure mathematics related to basic notions he or she uses in day-to-day work? In fact, given a concrete problem, this may be false today and true tomorrow.

The material presented here includes a variety of possibilities to learn notions and results of linear algebra. As online text, generated on demand, it includes – without lacking exactness – different levels of difficulty, presenting theorems under stronger assumptions for the beginner or in greater generality for an ambitious student.

Apparently, an upgrade of knowledge, acquired at the beginning, can be much easier achieved if you simply have to switch a button and obtain (with equal notations) a new hyper-text document written in a personalised way: Including more or less of the necessary basics, examples, additional comments, etc.

The material is accompanied by a collection of exercises, some of them generated with random initial values and – if requested – equipped with detailed solutions. The online-output is computed and assembled using the computer algebra - system SINGULAR [1].

Textbooks [2] and a CD-ROM edition of exercises [3] offer a complete basic course on linear algebra, which can be used independently, as well. Applying computer-algebra is suggested and trained here at an early state.

The material has been successfully tested at *Humboldt-Universität zu Berlin* for students of mathematics and informatics. Some parts (related to the collection of exercises) still have to be considered as work in progress.

## References

- [1] Greuel, G.-M., Pfister, G., Schönemann, H., *Singular Reference Manual*. Reports On Computer Algebra 12, May 1997, Center for Computer Algebra, University of Kaiserslautern (<http://www.singular.uni-kl.de/>).
- [2] Roczen, M., Wolter, H., *Lineare Algebra individuell*, vol. 1, 2. Lulu, Morrisville NC, 2005 (ISBN 1-4116-2648-6 and ISBN 1-4116-3558-2).
- [3] Roczen, M., Wolter, H., Pohl, W., Popescu, D., Laza, R., *Lineare Algebra individuell - Aufgabensammlung* (CD-ROM Fassung, Ver. 0.52), Lulu, Morrisville NC, 2006.

## Some features about the convergence process of a Genetic Algorithm

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Genetic Algorithms (GA) are inspired by nature, by the principle of evolution, i.e., survival of the fittest. GA are applicable to many hard optimization problems. The importance of these evolution programming techniques is growing since evolution processes are parallel in nature and parallelism is one promising direction in Computer Science.

The underlying idea of GA is extremely simple. Consider a population  $P$  of prey, with characteristics making them more or less likely to be eaten by predators and let us suppose that we can describe these features that permits an individual  $p$  to survive by some *fitness function*  $f$  i.e.: the higher the value of  $f(p)$ , the higher the probability of survival of  $p$  -the population, of course, evolves in time-. For obvious reasons, one expects the prey with high fitness to eventually dominate the population. This is exactly how a basic GA works. Formally, one may thus think of an optimization problem. In this review of Genetic Algorithms -and their convergence process- we do not aim at completeness. Our wish is to provide a brief overview that is broad enough to show the richness of this field (only occasionally fill in details but refer amply to existing literature): we will describe in a intuitive way what GA are about and we will briefly sketch some of the mathematics behind this (such that *the schema theorem* by Holland ([3]) and concepts like *deception* ([2]), (high) *epistasis* ([1], [4]) and (high) *order* ([4]) of the fitness function). Moreover, we give the main ideas of *Contractive Genetic Algorithms* (CGA). The *Banach fix point theorem* has an intuitive application to the case of GA. In fact, it may be proved the convergence of CGA to the same fix point independently of the choice of initial population. The fix point is achieved when all individuals in the population have the same -global maximum- value (see [5] for details).

## References

- [1] Davidor, Y., Epistasis variance: a viewpoint on GA-hardness. (ed. by G. J. E. Rawlins) *Foundations of Genetic Algorithms*, Morgan Kaufmann Publishers Inc., San Francisco, 1991, 23-25.

- [2] Goldberg, D. E., *Genetic Algorithms in Search, Optimization and Machine Learning*. Addison-Wesley, Reading, MA, 1989.
- [3] Holland, J. H., *Adaptation in Natural and Artificial Systems* 2nd edition. MIT Press, MA, Cambridge, 1992.
- [4] Iglesias, M. T., Naudts, B., Verschoren, A., Vidal, C., *Foundations of Generic Optimization. Volume 1: A Combinatorial Approach to Epistasis*. Springer, Netherlands, 2005.
- [5] Michalewicz, Z., *Genetic Algorithms + Data Structures = Evolution Programs*. 3rd edition, Springer-Verlag, Berlin Heidelberg New York, 1996.

## Learning Mathematics with Up-to-date Technology

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Our teaching will be more effective if we connect the reality and the student's interests with the practice and goal of the academic world. Likewise, it is necessary to integrate the Mathematics contents in a wide cultural frame in order to provide a meaningful teaching. Nowadays, with each computer, our student has a reference library, a laboratory, a calculator center, a printer and a design study among other possibilities. Definitely, the computer is the basic way of access and generation of knowledge ([1],[2]). Moreover, the students like it.

As an example, we have considered as focus of our experience the geometric curve named "lunulae". Around this concept we integrate mathematical, cultural, theoretical and practical contents by means of internet-based learning activities .

Specially, we have proposed several interesting points:

- Nature
- Archaeological Art
- The first study with Hipocrates of Chios
- Geometric properties
- Contributions of D. Bernouilli
- Object design

In order to organize the tasks and provide sufficient autonomy we designed a web-quest. Through the browser the student searched information on cultural interest points. The outcome of this work encouraged the following mathematical tasks. The solution of geometric problems was guided and obtained through a dynamic geometry program. With this, the students could experiment, approach, speculate and solve the problems. Finally, we proposed free design works that included lunulae, with this we expected the students to use geometric concepts and processes in a meaningful manner. The usual word processor was used to obtain a final report.

The experimentation was done with secondary students. The general good results were specially valuable in aspects such as the induced interests, the high autonomy and the meaningful use of geometry.

## References

- [1] Guzmán, M., Tendencias actuales en educación Matemática, *Textos de Miguel de Guzmán*, Monografía de Suma **2**, 79–108, Madrid, 2005.
- [2] Griffiths, P. A., Las Matemáticas ante el cambio de milenio, *La Gaceta de la R.S.M.E.* **3** (2000), 23–41.

## Monuments on mathematicians

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There is a collection of photographs on monuments commemorating to mathematicians in the internet [1]. The roots of this collection are in 2001 and meanwhile it possesses pictures about almost 100 location based memorials like monuments, plaques, museums, graves, tombs, busts, stones, temples, wells etc. as well as paintings and sculptures. Moreover it actually contains also some images of banknotes and of a couple of stamps. An additional section on coins is under preparation. The accompanying texts are usually provided in German as well as in English.

This poster presentation is bound to inform about the intention, the scope, the aims and the contents of the virtual exhibition. Certainly problems and difficulties should also be discussed. Besides some exemplary pictures a geographic overview of the original places of the exhibits will be provided. Additional remarks on the history and the development of this virtual exhibition will probably be of interest too.

The poster session is mainly designed to inform about the virtual exhibition but may also serve as a call for contribution.

## References

- [1] Monuments on mathematicians - a virtual exhibition.  
<http://www.w-volk.de/museum/exposi.htm>