Carl Friedrich Gauss Prize awarded to Kiyoshi Itô

The first laureate of the newly created Gauss prize for applications of mathematics is the Japanese mathematician Kiyoshi Itô, 90. The prize will be awarded at the opening ceremony of the International Congress of Mathematicians in Madrid on August 22, 2006.

Carl Friedrich Gauss (1777–1855), one of the greatest mathematicians of all times, is not only known for achievements in very abstract fields like number theory. He also created tools that serve the physicist as well as the engineer or everybody who wants to draw conclusions from measurements of all kinds, with their inevitable inaccuracies. They all use Gauss's "least squares method" to find the correct numbers behind huge amounts of unreliable data.

Mathematics is not just playing around with building blocks invented for that purpose. It has a profound impact to virtually all sciences and, more or less indirectly, to technology, business and everyday life. To improve the public awareness of this fact, the Gauss prize was created. The prize is awarded jointly by the Deutsche Mathematiker-Vereinigung (DMV = German Mathematical Union) and the International Mathematical Union (IMU), and administered by the DMV. It consists of a medal and a monetary award (currently valued at EUR 10,000). The source of the prize is the surplus from the International Congress of Mathematicians (ICM’98) held in Berlin. The prize will be awarded for the first time at this year's ICM in Madrid.

The prizewinner is the Japanese mathematician Kiyoshi Itô, aged 90, and the subject of his prize-honored work is doubtlessly connected to everyday life: it is chance, those tiny, unpredictable effects that decide which way a die falls or a roulette ball rolls. Of course, it is impossible to predict the unpredictable; nevertheless, you can do statistics to determine, e. g., the probability of getting three sixes in a row by tossing a die three times, the expected time until your complete ruin if you keep gambling in a casino, or, more seriously, the likelihood that a success in a new form of medical treatment is due to a new drug and not just to chance.

The kind of chance Itô worked on, however, is a particularly wild and pure one. Unlike in tossing a die where the unpredictability is confined to well-separated, "discrete" events, this kind of chance can strike at any time. The prototypic example is the so-called Brownian motion. Small pollen grains or dust particles exhibit an erratic motion that can be viewed under the microscope due to collisions with water molecules that are themselves invisible.

The mathematical model of this kind of motion is called a stochastic process. The random forces that keep the particle moving are blind and without memory; this means, they don't care for the actual position of the particle they are pushing around, and they don't even remember when they hit the last time. This is completely reasonable if you think of water molecules – how could they remember? –, but at the same time it renders a Brownian path a very difficult mathematical object. In technical terms, it is nowhere differentiable and its length is infinite.

Even those properties don't prevent you from doing basic statistics. So you can deduce that the expected distance of a Brownian particle from its initial position grows proportional to the square root of the time. But if random and classical (deterministic) forces act together or if you want to control the particle's path, e. g. to counteract its random movement, classical mathematical tools are bound to fail.

This was remedied by Kiyoshi Itô who, beginning in the 1940s, developed a completely new mathematical formalism named stochastic analysis. It allowed mathematicians to formulate that mixture of random and deterministic forces in a so-called stochastic differential equation and even to solve those equations, in a sense.

Itô's theory is sufficiently abstract to apply to fields that are completely different from the motion of dust in water. Stock prices on the financial market are subject to random forces not unlike those that act in a Brownian motion. Bankers who try to counteract the effects of those fluctuations find themselves forced to trade "in continuous time", at least in theory. Out of Itô's ideas grew a strategy for continuous trading and, eventually, a formula to calculate the price of an option. Today the Black-Scholes formula underlies almost all financial transactions that involve options or futures; moreover, it won two of its inventors the 1997 Nobel prize in economics.

Beyond particle positions and share prices, Itô's theory applies also to the size of a population of living organisms, to the frequency of a certain allele within the gene pool of a population, or even more complex biological quantities. Due to Itô's work, biologists can assess the probability with which a gene will dominate the whole population or a species will survive.

It took mathematicians themselves quite a while to appreciate the importance of Itô's results. This is partially due to Japan's isolation during World War II. Only from 1954 on, Itô lectured on his achievements at the Institute of Advanced Studies in Princeton.

Today, there is no doubt that stochastic analysis is a rich, important and fruitful branch of mathematics with a formidable impact to "technology, business, or simply people’s everyday lives".

Contact: Martin Groetschel, groetschel@zib.de. Tel: +49 (30) 3657329 (Any statement will be under embargo until August 22nd, CET).