Fields Medal
Wendelin Werner

CITATION:
"For his contributions to the development of stochastic Loewner evolution, the geometry of two-dimensional Brownian motion, and conformal field theory"

The work of Wendelin Werner and his collaborators represents one of the most exciting and fruitful interactions between mathematics and physics in recent times. Werner's research has developed a new conceptual framework for understanding critical phenomena arising in physical systems and has brought new geometric insights that were missing before. The theoretical ideas arising in this work, which combines probability theory and ideas from classical complex analysis, have had an important impact in both mathematics and physics and have potential connections to a wide variety of applications.

A motivation for Wendelin Werner's work is found in statistical physics, where probability theory is used to analyze the large-scale behavior of complex, many-particle systems. A standard example of such a system is that of a gas: Although it would be impossible to know the position of every molecule of air in the room you are sitting in, statistical physics tells you it is extremely unlikely that all the air molecules will end up in one corner of the room. Such systems can exhibit phase transitions that mark a sudden change in their macroscopic behavior. For example, when water is boiled, it undergoes a phase transition from being a liquid to being a gas. Another classical example of a phase transition is the spontaneous magnetization of iron, which depends on temperature. At such phase transition points, the systems can exhibit so-called critical phenomena. They can appear to be random at any scale (and in particular at the macroscopic level) and become "scale-invariant", meaning that their general behavior appears statistically the same at all scales. Such critical phenomena are remarkably complicated and are far from completely understood.

In 1982 physicist Kenneth G. Wilson received the Nobel Prize for his study of critical phenomena, which helped explain "universality": Many different physical systems behave in the same way as they get near critical points. This behavior is described by functions in which a quantity (for instance the difference between the actual temperature and the critical one) is raised to an exponent, called a "critical exponent" of the system. Physicists have conjectured that these exponents are universal in the sense that they depend only on some qualitative features of the system and not on its microscopic details. Although the systems that Wilson was interested in were mainly three- and four-dimensional, the same phenomena also arise in two-dimensional
systems. During the 1980s and 1990s, physicists made big strides in developing conformal field theory, which provides an approach to studying

two-dimensional critical phenomena. However, this approach was difficult to understand in a rigorous mathematical way, and it provided no geometric picture of how the systems behaved. One great accomplishment of Wendelin Werner, together with his collaborators Gregory Lawler and Oded Schramm, has been to develop a new approach to critical phenomena in two dimensions that is mathematically rigorous and that provides a direct geometric picture of systems at and near their critical points.

Percolation is a model that captures the basic behaviour of, for example, a gas percolating through a random medium. This medium could be a horizontal network of pipes where, with a certain probability, each pipe is open or blocked. Another example is the behaviour of pollutants in an aquifer. One would like to answer questions such as, What does the set of polluted sites look like? Physicists and mathematicians study schematic models of percolation such as the following. First, imagine a plane tiled with hexagons. A toss of a (possibly biased) coin decides whether a hexagon is colored white or black, so that for any given hexagon the probability that it gets colored black is p and the probability that it gets colored white is then 1 - p. If we designate one point in the plane as the origin, we can ask, Which parts of the plane are connected to the origin via monochromatic black paths? This set is called the "cluster" containing the origin. If p is smaller than 1/2, there will be fewer black hexagons than white ones, and the cluster containing the origin will be finite. Conversely, if p is larger than 1/2, there is a positive chance that the cluster containing the origin is infinite. The system undergoes a phase transition at the critical value p = 1/2.

This critical value corresponds to the case where one tosses a fair coin to choose the color for each hexagon. In this case, one can prove that all clusters are finite and that whatever large portion of the lattice one chooses to look at, one will find (with high probability) clusters of size comparable to that portion. The accompanying picture represents a sample of a fairly large cluster.

The percolation model has drawn the interest of theoretical physicists, who used various non-rigorous techniques to predict aspects of its critical behavior. In particular, about fifteen years ago, the physicist John Cardy used conformal field theory to predict some large-scale properties of percolation at its critical point. Werner and his collaborators Lawler and Schramm studied the continuous object that appears when one takes the large-scale limit---that is, when one allows the hexagon size to get smaller and smaller. They derived many of the properties of this object, such as, for instance, the fractal dimension of the boundaries of the clusters. Combined with Stanislav Smirnov's 2001 results on the percolation model and earlier results by Harry Kesten, this work led to a complete derivation of the critical exponents for this particular model.

Another two-dimensional model is planar Brownian motion, which can be viewed as the large-scale limit of the discrete random walk. The discrete random walk describes the trajectory of a particle that chooses at random a new direction at every unit of time. The geometry of planar Brownian paths is quite complicated. In 1982, Benoit Mandelbrot conjectured that the fractal dimension of the outer boundary of the trajectory of a Brownian path (the outer boundary of the blue set in the accompanying picture) is 4/3. Resolving this conjecture seemed out of reach of classical probabilistic techniques. Lawler, Schramm, and Werner proved this conjecture first by showing that the outer frontier of Brownian paths and the outer boundaries of the continuous percolation clusters are similar, and then by computing their common dimension using a dynamical construction of the continuous percolation clusters. Using the same strategy, they also derived the values of the closely related "intersection exponents" for Brownian motion and simple random walks that had been conjectured by physicists B. Duplantier and K.-H. Kwon (one of these intersection exponents describes the probability that the paths of two long walkers remain disjoint up to some very large time). Further work of Werner exhibited additional symmetries of these outer boundaries of Brownian loops.

Another result of Wendelin Werner and his co-workers is the proof of the "conformal invariance" of some two- dimensional models. Conformal invariance is a property similar to, but more subtle and more general than, scale invariance and lies at the roots of the definition of the continuous objects that Werner has been studying. Roughly speaking, one says that a random two-dimensional object is conformally invariant if its distortion by angle-preserving transformations (these are called conformal maps and are basic objects in complex analysis) have the same law as the object itself. The assumption that many critical two-dimensional systems are conformally invariant is one of the starting points of conformal field theory. Smirnov's above-mentioned result proved conformal invariance for percolation. Werner and his collaborators proved conformal invariance for two classical two-dimensional models, the loop-erased random walk and the closely related uniform spanning tree, and described their scaling limits. A big challenge in this area now is to prove conformal invariance results for other two-dimensional systems.

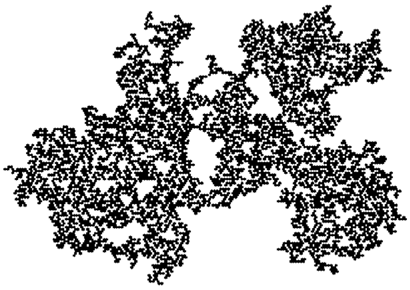
Mathematicians and physicists had developed very different approaches to understanding two-dimensional critical phenomena. The work of Wendelin Werner has helped to bridge the chasm between these approaches, enriching both fields and opening up fruitful new areas of inquiry. His spectacular work will continue to influence both mathematics and physics in the decades to come.

1357 words

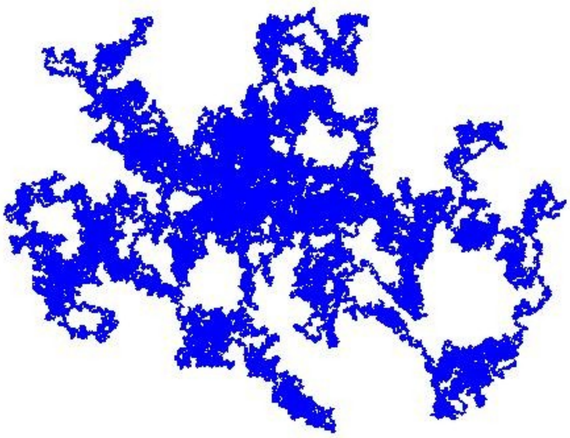
BIOGRAPHICAL SKETCH

Born in 1968 in Germany, Wendelin Werner is of French nationality. He received his PhD at the University of Paris VI in 1993. He has been professor of mathematics at the University of Paris-Sud in Orsay since 1997. From 2001 to 2006, he was also a member of the Institut Universitaire de France, and since 2005 he has been seconded part-time to the Ecole Normale Supérieure in Paris. Among his distinctions are the Rollo Davidson Prize (1998), the European Mathematical Society Prize (2000), the Fermat Prize (2001), the Jacques Herbrand Prize (2003), the Loève Prize (2005) and the Pólya Prize (2006).

PICTURES



Caption: A percolation cluster. Image courtesy of Wendelin Werner.



Caption: The path of Brownian motion. Image courtesy of Wendelin Werner.

Wendelin Werner portrait, courtesy of Wendelin Werner.